Stochastic acoustic control of internal sound using TMD

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ABSTRACT

The present work is intended to introduce a stochastic acoustic optimization of Tuned Mass Damper (TMD) parameters used to control the internal sound induced by random vibrations of a flexible structure coupled to a cavity filled with air. Assuming linear behavior of the entire vibro-acoustic system, the modal interaction approach can be used and the control, made in the low frequency range, can be performed considering the root mean square acoustic pressure (RMSAP) as the objective function to be minimized. Indeed, in presence of random excitation applied to the flexible structure, the acoustic pressure measured at a given location into the cavity can be characterized by its RMSAP. A spectral analysis has been made over different bandwidth values and the RMSAP is evaluated. Depending on the bandwidth parameter, used to evaluate the objective function, two kind of control has been defined: (1) the broadband control, corresponding to small values of the bandwidth parameter, and (2) the narrowband control corresponding to large values of the bandwidth parameter. The numerical investigations showed that for the coupled mode dominated by an in-vacuo structural mode, a broadband control allows obtaining satisfactory performance and the TMD has been acting as a dissipative device. In the opposite, for the coupled mode dominated by a rigid-walled cavity mode, a narrowband control is more efficient and the TMD has been acting as a highly reactive device.

1 INTRODUCTION

The potentialities of the TMD device in structural vibration mitigation [1] as well as in the internal sound control are recognized. The performance of such device strongly depends on its parameters and in the present work a new stochastic optimization strategy based on an acoustic criterion is presented. The strategy consists to find the optimum TMD parameters as well as the optimum location, minimizing the RMSAP measured at a given location into the cavity. The numerical investigations showed that the proposed strategy can deal with the two kind of coupled modes (i.e. those are dominated by the structure and those dominated by the cavity [2]).

2 GOVERNING EQUATIONS, THE OPTIMIZATION STRATEGY

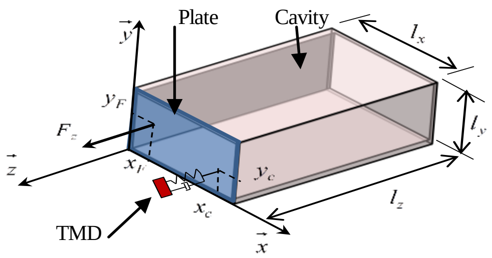

Figure 1. The TMD device attached to a flexible plate coupled to a cavity

Figure 1 shows a TMD with mass m_T, a natural frequency $\omega_T = \sqrt{k_T/m_T}$ and a damping ratio $\xi_T = c_T/2m_T\omega_T$, attached to a flexible plate (with a thickness h) coupled to a cavity, which is in turn, filled with air. The stationary zero mean white noise excitation F_z is applied at $\mathbf{r}_F = (x_F, y_F)^{\mathrm{T}}$ whereas the TMD is attached at $\mathbf{r}_c = (x_c, y_c)^{\mathrm{T}}$. The dimensions of the system are as shown in Figure 1. Let Ns and Na be the number of modes considered in the analysis for the structure and the cavity; by assuming linear behaviour and light proportional damping in the structure and the cavity [2, 3], the modal interaction approach [2] can be used and the governing equations can be written in modal coordinates as follows:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{\Phi}^{\mathrm{T}} F_z \tag{1}$$

where $\mathbf{q} = (\mathbf{w}^{\mathrm{T}}, \mathbf{p}^{\mathrm{T}}, z_T)^{\mathrm{T}}$, $\mathbf{w}$ and $\mathbf{p}$ are the (Ns×1) and the (Na×1) vectors of the modal participation factor, respectively; z_T is the displacement of the TMD; $\mathbf{\Phi} = (\boldsymbol{\varphi}_F \quad \boldsymbol{\psi}_0 \quad 0)$, $\boldsymbol{\varphi}_F$ is the (Ns×1) vector of the plate mode shapes computed at force location (x_F, y_F) and $\boldsymbol{\psi}_0$ is a (Na×1) vector of zeros; $\mathbf{M} = \begin{bmatrix} \mathbf{\Lambda}_m & \mathbf{0} & \mathbf{0} \\ S\mathbf{C}_{nm} & \frac{1}{\rho_0 c_0^2}\mathbf{\Lambda}_n & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & m_T \end{bmatrix}$, $\mathbf{\Lambda}_m = \begin{bmatrix} \ddots & & \\ & \Lambda_m & \\ & & \ddots \end{bmatrix}$, $\mathbf{\Lambda}_n = \begin{bmatrix} \ddots & & \\ & \Lambda_n & \\ & & \ddots \end{bmatrix}$;

$\mathbf{D} = \begin{bmatrix} \mathbf{D}_m + c_T \boldsymbol{\varphi}_c^{\mathrm{T}} \boldsymbol{\varphi}_c & \mathbf{0} & -c_T \boldsymbol{\varphi}_c^{\mathrm{T}} \\ \mathbf{0} & \mathbf{D}_n & \mathbf{0} \\ -c_T \boldsymbol{\varphi}_c & \mathbf{0} & c_T \end{bmatrix}$, $\mathbf{D}_m = \begin{bmatrix} \ddots & & \\ & 2\xi_m \omega_m \Lambda_m & \\ & & \ddots \end{bmatrix}$ $\mathbf{D}_n = \frac{1}{\rho_0 c_0^2} \begin{bmatrix} \ddots & & \\ & 2\xi_n \omega_n \Lambda_n & \\ & & \ddots \end{bmatrix}$; ρ_0 is the density of air, c_0 is the celerity of sound in air, Λ_m and Λ_n are the structural modal mass and the modal volumes of the cavity, respectively; $S = l_x \times l_y$; $\boldsymbol{\varphi}_c$ is the $(1 \times Ns)$ vector of the plate mode shapes calculated at TMD location (x_c, y_c) and ξ_m, ξ_n are the damping ratios of the plate and the cavity, for a given modes "*m*" and "*n*", respectively; $\mathbf{C}_{nm}$ is the coupling matrix [3]; $\mathbf{K}=\begin{bmatrix}\mathbf{K}_m+k_T\boldsymbol{\varphi}_c^{\mathrm{T}}\boldsymbol{\varphi}_c & -S\mathbf{C}_{nm}^{\mathrm{T}} & -k_T\boldsymbol{\varphi}_c^{\mathrm{T}}\\ \mathbf{0} & \mathbf{K}_n & \mathbf{0}\\ -k_T\boldsymbol{\varphi}_c & \mathbf{0} & k_T\end{bmatrix}$, $\mathbf{K}_m=\begin{bmatrix}\ddots & & \\ & \omega_m^2\Lambda_m & \\ & & \ddots\end{bmatrix}$, $\mathbf{K}_n=\frac{1}{\rho_0 c_0^2}\begin{bmatrix}\ddots & & \\ & \omega_n^2\Lambda_n & \\ & & \ddots\end{bmatrix}$. It can be noted that readers can refer to [2, 3] for further details about the model. Let $\tilde{\mathbf{q}}(\omega)$, $\tilde{\mathbf{w}}(\omega)$, $\tilde{\mathbf{p}}(\omega)$, $\tilde{z}_T(\omega)$ and $\tilde{F}_z$ be the finite Fourier transform of $\mathbf{q}$, $\mathbf{w}$, z_T and F_z respectively. The Fourier transform of both sides of Equation (1) yields to $\tilde{\mathbf{q}}=\left(-\omega^2\mathbf{M}+j\omega\mathbf{D}+\mathbf{K}\right)^{-1}\boldsymbol{\Phi}^{\mathrm{T}}\tilde{F}_z$. The modal acoustic pressure $\tilde{\mathbf{p}}(\omega)$ is given by $\tilde{\mathbf{p}}(\omega)=\mathbf{Y}\boldsymbol{\Phi}^{\mathrm{T}}\tilde{F}_z$, where $\mathbf{Y}$ is the $Na\times(Ns+Na+1)$ sub-matrix extracted from the matrix $\left(-\omega^2\mathbf{M}+j\omega\mathbf{D}+\mathbf{K}\right)^{-1}$ by taking the Na rows corresponding to the modal acoustic pressure $\tilde{\mathbf{p}}(\omega)$. The power spectral density (PSD) of the acoustic pressure at a given location $\mathbf{r}_a$ into the cavity and for a force location $\mathbf{r}_F$ can be expressed as follows $S_{\tilde{p}\tilde{p}}(\mathbf{r}_a,\mathbf{r}_F,\omega)=\left|H(\omega,\mathbf{r}_a,\mathbf{r}_F)\right|^2 S_{FF}$, where S_{FF} is the constant PSD of the white noise force applied to the plate and $H(\omega,\mathbf{r}_a,\mathbf{r}_F)=\boldsymbol{\psi}(\mathbf{r}_a)\mathbf{Y}(\omega)\boldsymbol{\Phi}^{\mathrm{T}}(\mathbf{r}_F)$; $\boldsymbol{\psi}(\mathbf{r}_a)$ is the $(1\times Na)$ vector of the acoustic mode shape calculated at location $\mathbf{r}_a$ (microphone location). Once the acoustic PSD response is obtained, the stochastic acoustic optimization strategy based on the RMSAP can be formulated as follows:

$$\text{Find } \mathbf{d}=(\omega_T,\xi_T,\mathbf{r}_c)^{\mathrm{T}} \text{ to minimize RMSAP}=\sigma_p(\mathbf{r}_a,\mathbf{r}_F,\mathbf{d})=\sqrt{\int_{\omega_1}^{\omega_2} S_{\tilde{p}\tilde{p}}(\mathbf{r}_a,\mathbf{r}_F,\mathbf{d},\omega)d\omega} \tag{2}$$

The evaluation of the objective function depends on the bandwidth $\Delta f=[\omega_1,\omega_2]$ therefore two kind of control have been defined: (1) the narrowband control, corresponding to small bandwidth parameter, and (2) the broadband control, corresponding to large bandwidth parameter. The performance of the TMD device will depend on this parameter.

3 NUMERICAL STUDY

The numerical values of the vibro-acoustic system are taken, $l_x=0.5\text{m}$, $l_y=0.3\text{m}$, $l_z=1.1\text{m}$, $h=3\text{mm}$; the plate has a Young's modulus $E=70\times10^9$ Pa , a density $\rho_s=2700\,\text{kg.m}^{-3}$ and a Poisson's ratio $\upsilon=0.3$; $\rho_0=1.21\,\text{Kg.m}^{-3}$, $c_0=344\,\text{m.s}^{-1}$, $Ns=21$ and $Na=102$. The mass of the TMD is taken 2% of the total mass of the plate; $S_{FF}=0.1\,\text{N}^2\times\text{Hz}^{-1}$, $\mathbf{r}_a=(0.35,0.1,-0.875)^{\mathrm{T}}$ and $\mathbf{r}_F=(0.05,0.05)^{\mathrm{T}}$. The frequency range of interest is taken [0, 220 Hz] where it has been observed two resonant coupled modes corresponding to $\varpi_1=108.59\text{Hz}$ and $\varpi_2=159.52\text{Hz}$. The first mode is dominated by the *in-vacuo* structural mode (1,1) whereas the second is dominated by the *rigid-walled* cavity mode (0,0,1).

Table 1 shows the optimum TMD parameters and their corresponding optimum locations for different bandwidth parameters Δf for the both resonant modes of interest. The inspection of the optimum damping ratios in Table 1 demonstrate that for the mode dominated by the *in-vacuo* structural mode (ϖ_1), the TMD device acts as a dissipative device whereas it acts as a highly reactive device when the mode dominated by the rigid-walled cavity mode (ϖ_2), is controlled. The obtained optimum TMD locations are always in the vicinity of the anti-node point of the *in-vacuo* structural mode (1,1). This result is predictable because both coupled modes (ϖ_1 and ϖ_2) involve the same *in-vacuo* structural mode (1,1).

In Figure 2 the PSD responses, for different Δf are presented when the two modes are separately controlled.

Targeted frequency	Δf (Hz)	ξ_T^*(%)	f_T^*(Hz)	x_c^*(m)	y_c^*(m)	σ_p^*(Pa)
$\varpi_1 = 108.59\text{Hz}$	2	0.010	110.897	0.240	0.149	0.003
	20	1.160	110.694	0.243	0.149	0.123
	40	13.45	110.919	0.253	0.151	0.328
$\varpi_2 = 159.52\text{Hz}$	2	0.693	155.554	0.192	0.147	0.008
	20	0.010	175.472	0.261	0.156	0. 125
	40	0.010	184.877	0.275	0.156	0.210

Table 1. Optimum TMD parameters for different bandwidth control.

The inspection of Figure 2 (a) shows that a reduction of 27.5 dB can be reached when a broadband control ($\Delta f = 40\text{Hz}$), of the targeted frequency ϖ_1, is achieved. A narrowband control produces two others peaks and the performance of the TMD device is less desirable. In Figure 2 (b), it's shown that a narrowband control ($\Delta f = 2\text{Hz}$) allows good performance of the TMD where a reduction of 20.6 dB is reached.

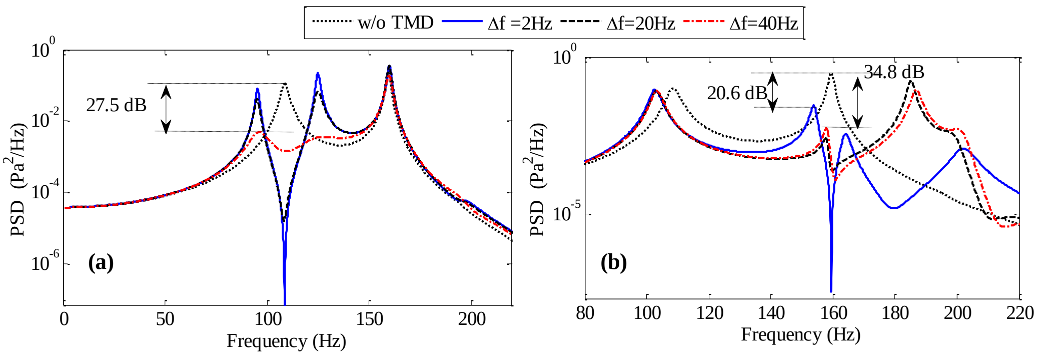

Figure 2. PSDs responses for different bandwidth parameters; (a): $\varpi_1 = 108.59\text{Hz}$, (b): $\varpi_2 = 159.52\text{Hz}$

4 CONCLUSIONS

In the present work a stochastic acoustic optimization strategy of TMD parameters is presented in order to control, in the low frequency range, the internal sound induced by stochastic vibrations of a flexible structure. The obtained results showed that for the coupled modes dominated by the *in-vacuo* structural mode, a broadband control is suitable to obtain satisfactory performance of the TMD, which has been acting as a dissipative device. In the opposite, for the mode controlled by the rigid-walled cavity mode, a narrowband control is more efficient in the internal sound control, where the TMD acted as a highly reactive device.

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