CONTINUOUS DESCRIPTION FOR THE DYNAMIC BEHAVIOUR OF 1D FRAMED STRUCTURES

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ABSTRACT

This work deals with the longitudinal vibration and transverse vibration of a specific 1D periodic framed structure, whose unit cells are interconnected beams. The associated dynamic behaviour will be investigated by the numerical Condensed Wave Finite Element method (CWFE) and the analytical Homogenization method of Periodic Discrete Media (HPDM). Homogenized models are deduced by the HPDM, while the numerical results obtained by CWFE serve as the reference to validate these models. Dispersion curves are presented to evaluate the valid frequency range of these models.

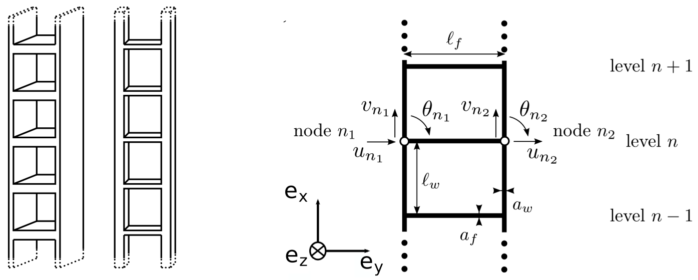
1 INTRODUCTION

The framed materials are widely employed in various industries, such as aeronautics (lattice beams), civil engineering (buildings), materials science (mechanics of foam and glass wool) and biomechanics (vegetable tissue or bones). Numerous methods (numerical and analytical) aiming to find their dynamic behaviors have been developed. Among the numerical methods, the most widely used is the Wave Finite Element Method (WFEM). Based on Floquet-Bloch theorem, WFEM employs the conventional finite element models of the unit cell to deduce the dynamics of the whole structure [1, 2]. As for the analytical method, one frequently discussed approach is the homogenization theory. In order to find an appropriate analytical continuous description for the periodic structures, several homogenization approaches have been developed, such as material receptance method, asymptotic expansion method, and the homogenization of periodic discrete media (HPDM) [3–5].

In this work, both analytical HPDM and numerical CWFE are employed to study a periodic discrete framed structure. The principal objective of this work is to re-evaluate the validity of the HPDM using the wave characteristics identified by CWFE.

2 TECHNIQUES AND RESULTS

The studied structure is a 'ladder', which is formed by a large number of unbraced beams, Figure 1. These identical beam unit cells follow the Euler-Bernoulli theory, and they are linked by perfectly stiff and massless nodes.



(a) Example (b) Notations

Figure 1. Studied structures [5]

2.1 HPDM

The HPDM is composed of two parts: discretization and homogenization. As the studied structure is made of interconnected beams, the dynamic balance of the whole structure can be expressed in a discrete form using the element balance and nodal balance. Thus, the kinematic description of the structure can be described by the motions of the nodes. Then, the scale separation being satisfied, the dynamic variables of neighbouring nodes can be connected by Taylor's Series, and the discrete dynamic variables at each node can be considered as specific values of a continuous function. For more details, please refer to [3–5].

According to the HPDM, the homogenized models for the longitudinal vibration is:

$$\Lambda\omega^2 V + 2E_w A_w V'' = 0 \tag{1}$$

And the homogenized models for the transverse vibration is:

$$\frac{2E_w^2 I_w I}{K} U'''''' - (2E_w I_w + E_w I)U'''' - \frac{E_w I}{K}\Lambda\omega^2 U'' + \Lambda\omega^2 U = 0 \tag{2}$$

Where V is the mean longitudinal displacement, U is the mean transverse displacement, Λ is the linear mass of the cell, and $K^{-1} = K_w^{-1} + K_f^{-1}$ is the shear stiffness of the cell, with $K_w = 24E_wI_w/l_w^2$, $K_f = 12E_fI_f/(l_wl_f)$;

2.2 CWFEM

The CWFEM is a combination of WFEM and mode reduction technique. Here, the fixed interface component mode synthesis method, or the Craig-Bampton method, is chosen to reduce mode order and speed up the calculation. Then, the method begins with establishing the motion equation of the unit cell, where the mass and stiffness matrices $\mathbf{M}$ and $\mathbf{K}$ can be extracted from conventional FE packages. After the reduction, the physical DOFs are then reformulated to a reduced modal basis of modal DOFs. And the following process is the same as WFEM. More details are shown in [6].

2.3 Dispersion curves

Here is an example structure, whose characteristics are listed in Table 1. To ensure the convergence of the mesh, all the beams are discretized into 20 finite elements. Each unit cell contains 183 DOFs, among which 171 are internal DOFs. After the reduction, the first 20 fixed interface modes are conserved.

l_w (m)	l_f (m)	a_w (m)	a_f (m)	ρ (kg m^{-3})	E (GPa)	μ
3	3	0.1	0.1	7600	2e11	0.3

Table 1. Material Properties

By considering the propagative waves in positive-x direction, the dispersion relation obtained by CWFEM is given in Figure 2. And the associated wave shape is plotted in Figure 3. According to the wave shapes, the first wave corresponds to the transverse vibration and the second wave appears to be the longitudinal vibration. A third wave shows up at about 10Hz. This is an atypical gyration mode which can not be predicted by HPDM. Thus, the first two modes are investigated and the comparison of dispersion curves obtained by CWFEM and HPDM are illustrated in figure 4.

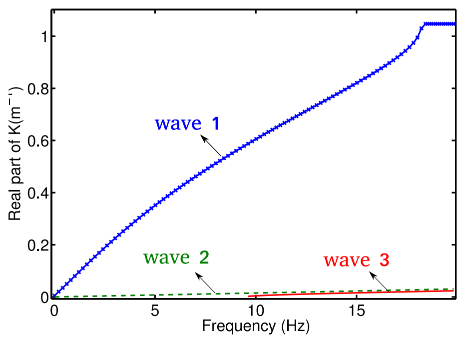
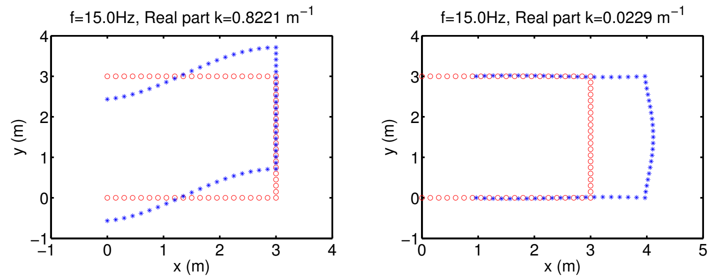


Figure 2. The dispersion relation from 0-20 Hz

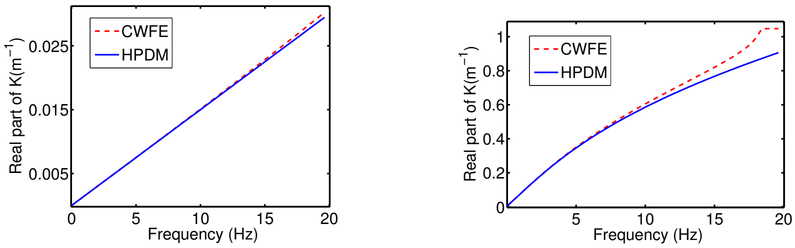
3 CONCLUDING REMARKS

The wave propagation feature of the 1D framed structure is studied through the dispersion relation obtained by both HPDM and CWFEM. Good agreement between the two results is an



(a) Wave 1 (b) Wave 2

Figure 3. Wave shapes (∗) Undeformed unit cell (o)



(a) The wave 1 (b) The wave 2

Figure 4. Dispersion relations by CWFEM and HPDM

evidence that the homogenized model achieves a reasonable accuracy. And the valid frequency range is limited in the first propagating zone.

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