ENERGY HARVESTING IN A NONLINEAR HARVESTER UNDER MODULATED DELAY AMPLITUDE

Zakaria Ghouli[1], Mustapha Hamdi[2], Faouzi Lakrad[1] and Mohamed Belhaq[1]

[1]Faculty of Sciences Ain Chock, University Hassan II-Casablanca, Morocco
Email: ghoulizakaria@gmail.com, lakrad@hotmail.com, mbelhaq@yahoo.fr

[2]FST-Al Hoceima
University Mohammed I Oujda, Al-Hoceima, Morocco
Email: hamustapha2000@yahoo.fr

ABSTRACT

We explore periodic and quasi-periodic (QP) vibration-based energy harvesting (EH) in a delayed nonlinear oscillator in which time delay feedback is inherently present in the system. The EH system consists in a delayed Duffing-van der Pol oscillator coupled to an electric circuit through an electromechanical coupling mechanism. We assume that the delay amplitude is modulated around a mean value with a certain frequency, and we consider the case of delay parametric resonance for which the frequency of the modulation is near twice the natural frequency of the oscillator. Application of the double-step perturbation method enables the approximation of the amplitude of the QP vibrations which is used to extract power from the harvester device. Results show that for small values of unmodulated delay amplitude, only the periodic vibration can be used to extract energy, while for larger values of unmodulated delay amplitude the periodic solution turns to unstable and only QP vibration can be used to extract energy with better performance. Numerical simulation is conducted to support the analytical predictions.

1 INTRODUCTION AND MODEL DESCRIPTION

In EH systems, the limitation of the linear attachment has been overcome by considering nonlinear stiffness in the mechanical part of the harvester. In this case the EH capability is improved [1, 2]. However, the EH performance provided by nonlinear attachments can suffer from instabilities and jump phenomena near the boundaries of the stable branch of the frequency response. To circumvent such instabilities, the idea of using QP vibrations is proposed to extract energy from delayed self-excited harvester systems [3]. The concept of using delayed feedback vibration absorber has also been used to enhance EH capability [4]. The purpose of the present work is to study the EH performance in a Duffing-van der Pol-type harvester device in which time delay is inherently present in the operating system, as in milling and turning operations [5]. The energy harvesting system consists then in a delayed Duffing-van der Pol oscillator coupled to an electric circuit through an electromechanical coupling mechanism; see Fig. 1.

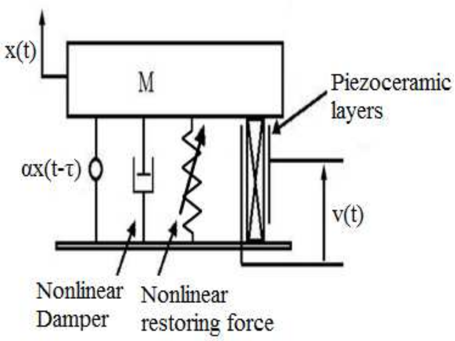

Figure 1. Schematic description of the EH system

The governing equation for the harvester system can be written in the dimensionless form as

$$\ddot{x}(t) + \delta\dot{x}(t) + \lambda\dot{x}(t)x(t)^2 + \omega_0^2 x(t) + \gamma x(t)^3 - \chi v(t) = \alpha x(t-\tau) \tag{1}$$

$$\dot{v}(t) + \beta v(t) + \kappa\dot{x}(t) = 0 \tag{2}$$

where $x(t)$ is the relative displacement of the rigid mass M, $v(t)$ is the voltage across the load resistance, δ and λ are the mechanical damping ratio, γ is the stiffness parameter, χ is the piezoelectric coupling term in the mechanical attachment, κ is the piezoelectric coupling term in the electrical circuit, β is the reciprocal of the time constant of the electrical circuit, α and τ are, respectively, the feedback gain and time delay. In this study we assume that the delay amplitude α is modulated around a mean value such that:

$$\alpha = \alpha_1 + \alpha_2\cos(\omega t) \tag{3}$$

where α_1 is the unmodulated delay amplitude and α_2, ω are, respectively, the amplitude and the frequency of the modulation. Note that the case where the nonlinear stiffness is absent (γ= 0) has been explored in [3] and the case of linear damper and unmodulated time delay was studied in [6].

2 MAIN RESULTS

We investigate the response of the system near the delay parametric resonance for which the frequency of the delay modulation is near twice the natural frequency of the oscillator. Application of the double-step perturbation method [7] enables the approximation of the amplitude of the QP vibrations which is used to extract power from the harvester device.

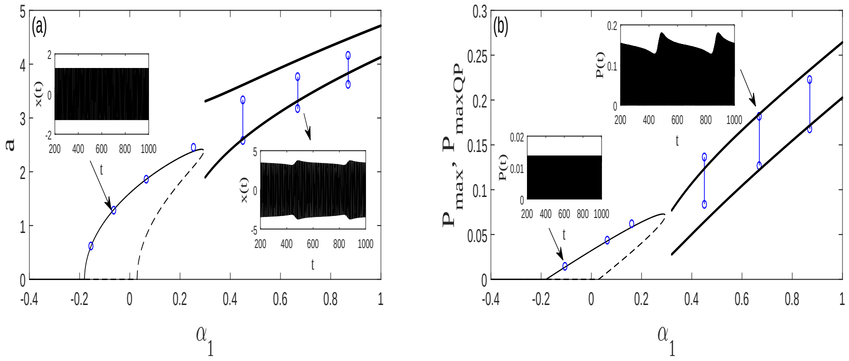

Figure 2: Vibration (a) and powers (b) amplitudes vs α_1 for $\alpha_2 = 0.25$, $\omega = 2$, $\chi = 0.05$, $\beta = 0.05$, $\lambda = 0.2$, $\delta = -0.1$, $\gamma = 0.05$, $\kappa = 0.5$, $\omega_0 = 1$ and $\tau = 5.2$. Analytical prediction (solid lines for stable and dashed line for unstable) and numerical simulation (circles).

In Fig. 2 is shown the variation of the amplitude of the periodic and the QP responses as well as the maximum output power amplitudes (P_{max}, P_{maxQP}) versus the unmodulated delay amplitude α_1 and for $\alpha_2 = 0.25$. The boxes inset in the figures show time histories of the amplitudes (Fig. 2a) and the power responses (Fig. 2b). It can be seen that for a small values of delay amplitude α_1, only the periodic vibration-based EH can be extracted. On the other hand, for relatively increasing value of α_1, the stable periodic solution disappear via saddle-node bifurcation, while energy can be extracted from QP vibration with better performance comparing with the periodic output power.

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