

INVERSE CHARACTERIZATION OF SANDWICH STRUCTURES USING SINGLE-SHOT WAVE SPEED MEASUREMENTS

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ABSTRACT

We propose an innovative method for the local inverse characterization of sandwich panels using a simple experimental procedure based on single shot pulse measurements. The method exploits a typical wave conversion phenomenon called bending-to-shear transition, producing a local energy velocity maximum within the transition bandwidth. Analytic expressions are derived for the first wave transition and used to retrieve skin's tensile modulus and equivalent honeycomb core's shear modulus from the transition frequency and the maximal group velocity. A simple iterative procedure is described to identify the transition frequency by comparison of the time delays between the pulse source and single measurement point. The group velocity can then be advantageously estimated using First Phase Arrival (FPA) times. A carbon fiber-reinforced sandwich panel with Nomex honeycomb core is considered for case study. The proposed method exhibits considerably reduced measuring and post-processing times, while experimental results are in good agreement with static measurements and the results from the Inhomogeneous Wave Correlation (IWC) method.

1 INTRODUCTION

Modelling sandwich structures in vibroacoustic environments is a challenging task due to the difficulty to perform accurate models of composite structures in the medium frequency range. Although the literature is abundant on mechanical characteristics of sandwich panels, the material properties of a sandwich structure can change significantly depending on the manufacturing process, or the panel location considered. It is therefore crucial to perform a thorough characterization, rather than rely on material values reported in literature. Besides, the values obtained using static measurements can result in considerable discrepancies in the vibroacoustic range. In a context of increasing need for reliable and cost-effective characterization techniques, a number of developments were made for sandwich structures. One can cite the work of Karakoc and Freund [1] on the experimental determination of the compliance matrix of Nomex structures, or the identification technique proposed by Matter et al. [2] for evaluating the elastic and damping properties of sandwich laminates with soft cores from modal analyses. In composites, several studies have been conducted on the use of high-frequency waves to retrieve elastic constants from energy velocity measurements [3] with remarkable accuracy. However, the rapid development of the so-called "meta-structures", involving stiffened or locally resonant components distributed along periodic patterns yields considerable challenges in terms of dispersion analyses, hence for the use of these high-frequency techniques. Model-based inverse identification of the equivalent material properties in medium frequencies was also investigated. The Inhomogeneous Wave Correlation (IWC) method was developed for the identification of the dispersion curves. Material estimation is based on the wavenumber measurement technique developed by McDaniels and Shepard [4]. These methods however require to measure and post-process the entire displacement field to extract the k-space, leading to expensive and time-consuming characterizations. This paper presents a wave-based characterization technique for sandwich panels in the medium frequencies based on single-shot measurements [5], where complex scattering effects can be avoided. It combines the advantage of involving a simple experimental set-up with the need for local or in-situ characterizations procedures.

2 TRANSITION ANALYSIS: THEORY AND SIMULATIONS

Consider the propagation of flexural waves in a symmetric sandwich plate along the in-plane direction θ , where k is the directional wavenumber $k_{\theta} = f(\mu, S_{\theta}, D_{\theta})$. Assuming the rotation inertia is negligible compared with the shear effects and the bending stiffness per unit width of the skins are small in comparison with the one of the plate, the dispersion relation becomes:

$$k^{2} = \frac{\mu}{2S} \left(\omega^{2} + \omega \sqrt{\omega^{2} + \frac{4S^{2}}{\mu D}} \right)$$
(1)

where $S = h_c G_c \left(1 + \frac{h_s}{h_c}\right)^2$ and $D \simeq E_s h_s \left(\frac{h_c^2}{2} + h_c h_s + \frac{2h_s^2}{3}\right)$ are the transverse shear rigidity and bending stiffness of the plate, μ is the mass per unit area, while $h_s h_c G_c$ and E_s denote the skin's and core's thickness, shear coefficient and Young modulus, respectively. The group velocity reaches a local maximum when $\frac{\partial c_g}{\partial \omega} = 0$ admits a solution, resulting in the following equation:

$$\frac{\omega^2 \Omega - \Omega^3 + 2\omega_0^2 \omega}{\left(\omega^2 \Omega + \Omega^3 + 2\omega_0 \omega^2 + 2\omega^3\right) \sqrt{\frac{\mu}{2S}(\omega\Omega + \omega^2)}} = 0$$
(2)

The solution $\omega_T = \frac{2S}{\sqrt{3\mu D}}$ provides a definition of the so-called transition frequency. Note that the maximal velocity, $c_{max} = \frac{4}{3}\sqrt{\frac{2S}{3\mu}}$ is independent of the bending stiffness *D* of the plate.

Material	Thickness	Density (kg.m ⁻¹)	Young modulus (GPa)	Shear modulus (MPa)
Skin	0.3 mm	1451	$E_{s} = 81$	$G_s = 2780$
Core	25 mm	53	$E_c = 5.23$	$G_c = 50$

Table 1: Dimensions and material characteristics of the sandwich beam. Note that the core is described as an homogeneous medium. The beam's width is $l_y = 10$ mm and the damping is not considered in this model.

A numerical case-study is considered for the sandwich waveguide described in Table 1. The group velocity is estimated using tone burst signals of amplitude $U_0 = 1 \ \mu \text{m}$ involving n_0 cycles at the frequency f_0 with Hanning window defined for $t \le \frac{n_0}{f_0}$ by: $U_0 \sin\left(\frac{\pi f_0 t}{n_0}\right) \sin(2\pi f_0 t)$

$$u(t) = U_0 \sin\left(\frac{\pi f_0 t}{n_0}\right) \sin\left(2\pi f_0 t\right)$$
(3)

Noteworthy, the velocities shown in Figure 1 are in very good agreement with analytical results. The accuracy is good, considering the frequency spectrum bandwidth of the excitation signals. Using the proposed wave speed measurements results in a 2.1% overestimation of the shear *S* coefficient. The transition is observed at 1.8 kHz at the velocity $c_{max} = 831.6 \text{ m.s}^{-1}$. This yields:

$$E_{\text{trans}} = 81.16 \,\text{GPa} \quad \text{and} \quad G_{\text{trans}} = 50.04 \,\text{MPa}$$
 (4)

which is is very good agreement with the exact material parameters shown in Table 1.



Figure 1: (a) Transient FEM simulation of a short monochromatic wave pulse, pulsation is 4000 Hz, propagation is considered along 3 m. (b) Dispersion curves of the sandwich waveguide: comparison with FEM simulations using group velocity measurements.

3 EXPERIMENTAL RESULTS

This method based on transition and maximum velocity measurements is tested on a $60 \text{ cm} \times 288 \text{ cm}$ sandwich panel made of a 10 mm-thick Nomex honeycomb core involving a 3.2 mm cell size, while propagation is considered in the W-direction. The core is surrounded by 0.6 mm-thick Hexforce skins with multi-axial carbon-reinforced fibres mixed with SR1700 epoxy resin.

The density of the skins is $\rho_s = 1451 \text{ kg.m}^{-3}$ and the core's density is measured using the overall panel's weight $\rho_c = 99 \text{ kg.m}^{-3}$ (manufacturer: 96 kg.m⁻³). The experimental set-up consists in a shaker producing wave pulses and a measurement point located at 1 m from the excitation. Results from the Inhomogeneous Wave Correlation (IWC) method and the proposed characterisation strategy are compared in Table 2 with the manufacturer's data based on static measurements on a different sample. It shows a very good accuracy compared with other wave-based method, considering that the proposed method only involves two measurement points.

Parameter	IWC method	Manufacturer	Proposed method
	(67 points)	(static)	(2 points, ToF)
E (GPa)	62	70	69.8
G (MPa)	37.8	[30 - 38]	36.5

Table 2: Comparison of the estimated parameters obtained by two wave-based methods and manufacturer's values.

4 CONCLUDING REMARKS

To summarize, the paper presents a definition for the transition phenomenon using the existence of a local energy velocity maxima within the wave transition bandwidth and an in-situ procedure for measuring effective mechanical parameters of sandwich plates in the vibroacoustic range, described in Figure 2.



Figure 2. Experimental procedure summarized as a flow diagram.

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