WAVE CONVERSION PROCESS IN LIGHTWEIGHT STRUCTURES: DIFFUSION THROUGH DEFECTS IN THE TRANSITION BANDWIDTH

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ABSTRACT

Structural waveguides involving heterogeneous cross-sections or based on periodic patterns are often subjected to wave conversion phenomena. This paper is concerned with a specific type of conversion observed in sandwich structures called bending-to-shear conversion. It occurs in the "transition" bandwidth, where the flexural wave is partially localized in the core of the sandwich, hence mainly governed by its shear modulus. This conversion has consequences on the wave reflection characteristics through a defect's interface. The need for wave-based Structural Health Monitoring strategies providing sub-wavelength detection capabilities has been increasing with the development of advanced, often periodic lightweight components. It is therefore advantageous to predict and take into account conversion effects at the earliest design stage of SHM systems. This paper explores the consequences of such conversions on the reflection coefficient of flexural guided waves in lightweight structures. A Diffusion Matrix Method (DMM) is employed to estimate the diffusion of guided waves obtained using refined unit-cell FE models. Case-study is a composite sandwich waveguide with honeycomb core. Results show significant variations of the wave's sensitivity to small-scaled core defects and delaminations during the conversion process.

1 INTRODUCTION

Sandwich composites are widely use in aerospace and transportation industries. Their main advantage, apart from their exceptional strength-to-weight ratio, is their wide range of possible configurations. The variety of sandwich panels involving innovative core's geometries or skin laminates developed every year illustrates this growing interest in the design of structurally advanced lightweight components. Wave-based inspection techniques are usually exploiting Lamb waves for their ability to travel long distances with low attenuation and their predictable dispersion curves. Yet, wave conversions are commonly encountered in heterogeneous structures, especially at high frequencies. Literature is abundant with examples of wave conversion, steering or localisation effects appearing in sandwich structures. Recently, Putkis et al. [1] investigated the influence of various Lamb waves conversions in CFRP plates for practical NDE and SHM applications. Indeed, this phenomenon produces a modification of the strain energy distribution along the waveguide's cross-section, or in its unit-cell when the waveguide is based on a periodic pattern. For such structures, the Wave Finite Element Method (WFEM) is often used to perform broadband wave dispersion analyses and understand wave's physics in complex periodic waveguides.

The proposed study focuses on a phenomenon occurring in the so-called first transition bandwidth [2, 3], which is a specific type of wave conversion appearing in sandwich structures subjected to flexural vibrations. It usually occurs in the low- or medium-frequency range, where local resonances and Bragg scattering effects are not observed. This conversion can easily be determined from the dispersion curves, as the passage from a behavior where flexural wave is governed by the skins' stiffness to one governed by the core's transverse shear. The correlation between this conversion process and the sensitivity to localized defects can therefore be conducted using a spectral diffusion analysis based on a FEM description of the interface between the healthy and damaged waveguides.

2 DIFFUSION MATRIX METHOD

The waveguide is considered as a straight elastic structure made of N identical substructures of same length along the main direction x. The wave dispersion characteristics can be derived from Bloch's theorem, denoting λ the propagation constant relating the displacements $\mathbf{q_n}$, $\mathbf{q_{n+1}}$ between two cells, by solving the spectral problem:

$$\mathbf{S}(\lambda,\omega) = (\lambda \mathbf{D}_{LR} + (\mathbf{D}_{LL} + \mathbf{D}_{RR}) + \frac{1}{\lambda}\mathbf{D}_{RL})\mathbf{q_n} = \mathbf{0}, \tag{1}$$

where $\mathbf{D}_{ij}$ are the dynamic stiffness matrices related to the left and right degrees of freedom of the unit cell's governing equation. The displacement $\mathbf{q_n}$ of any substructure n can be written $\mathbf{q_n} = \mathbf{\Phi Q}_n$ using the wave solutions of Eq.(1), where the incident 'inc' and reflected 'ref' wave amplitudes can be distinguished $\mathbf{Q}_n = \left[(\mathbf{Q}_n^{\text{inc}})^T, (\mathbf{Q}_n^{\text{ref}})^T\right]^T$ and the wave components are written:

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi}_q^{\text{inc}} & \mathbf{\Phi}_q^{\text{ref}} \\ \mathbf{\Phi}_F^{\text{inc}} & \mathbf{\Phi}_F^{\text{ref}} \end{bmatrix}, \tag{2}$$

The coupling element describing the junction between the healthy (1) and damaged (2) waveguides is described using a classical FEM, as shown in Figure 1. The reflection coefficient is derived from the dynamic equation of the condensed coupling element $\mathbb{D}^c$, resulting in the following scattering problem:

$$\begin{pmatrix} \mathbf{Q}^{\text{ref (1)}} \\ \mathbf{Q}^{\text{ref (2)}} \end{pmatrix} = \mathbb{C} \begin{pmatrix} \mathbf{Q}^{\text{inc (1)}} \\ \mathbf{Q}^{\text{inc (2)}} \end{pmatrix} \tag{3}$$

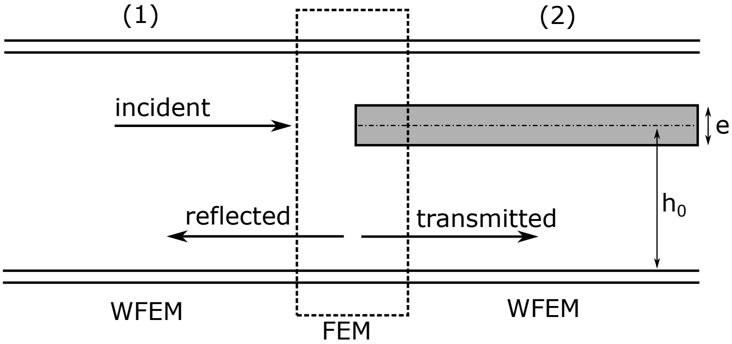

Figure 1. Description of the FEM/WFEM coupling problem for the diffusion analysis.

where $\mathbb{C}$ is the diffusion matrix written:

$$\mathbb{C} = -\left[\mathbb{D}^c \begin{bmatrix} \mathbf{\Phi}_q^{\text{ref}\,(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_q^{\text{ref}\,(2)} \end{bmatrix} + \begin{bmatrix} \mathbf{\Phi}_F^{\text{ref}\,(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_F^{\text{ref}\,(2)} \end{bmatrix}\right]^{-1} \times \left[\mathbb{D}^c \begin{bmatrix} \mathbf{\Phi}_q^{\text{inc}\,(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_q^{\text{inc}\,(2)} \end{bmatrix} + \begin{bmatrix} \mathbf{\Phi}_F^{\text{inc}\,(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_F^{\text{inc}\,(2)} \end{bmatrix}\right] \quad (4)$$

The incident wave is a first-order flexural wavetype i with normalized amplitude along the positive x-direction in waveguide (1) and the coefficient considered in matrix $\mathbb{C}$ corresponds to the $i \to i$ reflection, while the converted wavetypes $i \to j \neq i$ are not taken into account. A recent discussion of the definition of the reflection and transmission coefficients provided by the DMM can be found in [4].

3 REFLECTION OF FLEXURAL WAVE DUE TO CORE DEFECTS

The sandwich structure involves composite skins of thickness $h_s = 0.5$ mm, density $\rho_s = 1451$ and tensile modulus $E_s = 45$ GPa while the core is an homogeneous honeycomb medium of thickness $h_c = 15$ mm, density $\rho_c = 35$ and equivalent shear modulus $G_c = 80$ MPa. Poisson ratio is $\nu = 0.35$ and the unit-cell dimensions are $d_x = d_y = 1$ mm. The defect is defined by a vertical location $z_0 = 8$ mm in the core, a thickness e. It is modelled as a reduced core's stiffness at the defect's location: $\tilde{G} = G/r$. The transition frequencies are defined as the local maximum and minimum of the group velocity. These frequencies are used to defined the bandwidth where shear motion is the predominant behaviour.

Different damage severities defined by the value of e are compared in Figure 2.a. It shows a clear increase of the reflection coefficient between the two transitions, while the maximal amplitude of the reflection increases as expected with the damage thickness. Noteworthy, for the largest defect, the maximum reflection is twice the value of the minimum reflection in the bandwidth $[2\omega_T, 6\omega_T]$. The influence of the damage severity, defined using the reduction coefficient r is shown in Figure 2.b, where reflection coefficients are normalized to their maximum in the transition bandwidth. A similar phenomena can be observed since the maximum reflection is shifted from the second transition to the first as the defect severity increases. Results also indicate the expected reduction of the reflection above the second transition, which corresponds to wave localization in the skins. Higher sensitivities will therefore be obtained for crack and other skin's damages.

4 CONCLUSIONS

A local increase of the flexural wave reflection coefficient was identified within the transition bandwidth, associated with a bending-to-shear conversion. This result was found for different types of sandwich configurations, and is consistent with other investigations describing a partial localisation of strain energy within the transition bandwidth. It is emphasized that higher frequencies will always yield increased sensitivity due to wavelength reduction, but are therefore subjected to significant spatial attenuation and may exhibit complex scattering effects due to the periodicity of the waveguide. It is also mentioned that reflection coefficients are used in this study, since the *transmission* cannot be defined as such between two different waveguides.

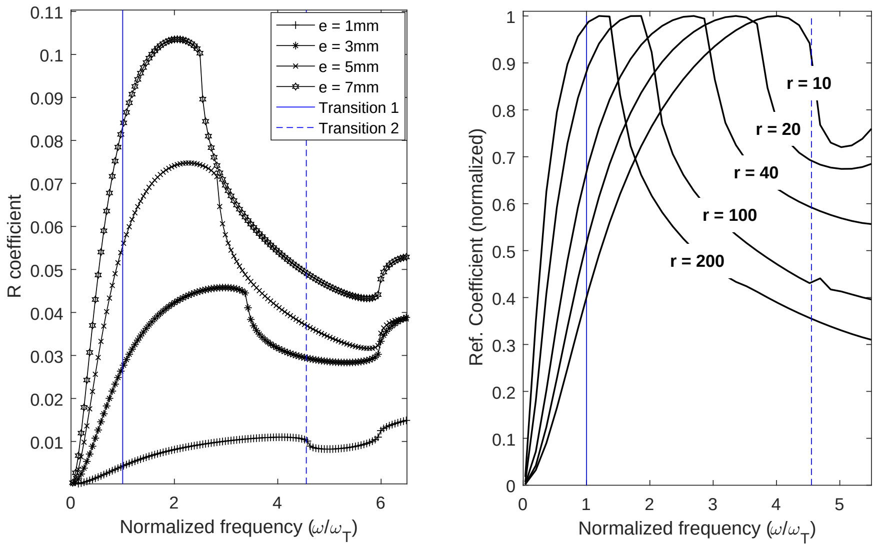

Figure 2: (a) Influence of the defect thickness on the reflection ($r = 10$). (b) Effect of the damage severity on the local maximum of the reflection ($e = 1$ mm).

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