STOCHASTIC ANALYSIS OF NEAR-PERIODIC COUPLED PENDULUMS CHAIN

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ABSTRACT

It is known that, when the mechanical coupling between the components is weak, small imperfections in a periodic structure can induce vibration localization. Stochastic analysis of near-periodic coupled pendulums chain is discussed in this paper. Perfect periodicity of the system is disturbed by varying randomly the length of one of the pendulums which is considered as an uncertain parameter. Its randomness is modeled in a probabilistic framework by a random variable according to a given range of dispersion level. Stochastic effects on vibration localization in mistuned four coupled pendulums chain is investigated through statistical evaluations. To do so, the propagation of uncertainties is performed using the Latin Hypercube Sampling method.

1 INTRODUCTION

Mistuning, or disorder, resulting from material defects, structural damage, manufacturing defaults, etc., breaks the perfect arrangement of periodic structures and alters significantly their dynamic behavior. The structure then becomes nearly periodic or called mistuned and vibration localization could occur under certain circumstances [1]. Zhu et al. [2] studied localization in randomly disordered coupled beams and proved that the wave propagation and localization can be altered by properly adjusting the structural parameters. Recently, Malaji et al. [3] investigated the effect of mistuning on vibration localization in two coupled pendulums chain. The main purpose of the present study is to investigate the stochastic effects of uncertain mistuning on vibration localization in a coupled pendulums chain.

2 MODEL

The scheme of N coupled pendulums chain is illustrated in figure 1. The pendulums have same mass m, torsional stiffness k_r and proportional damping constant c and are weakly coupled by translational springs k_t . An external base excitation x_g is applied to the system.

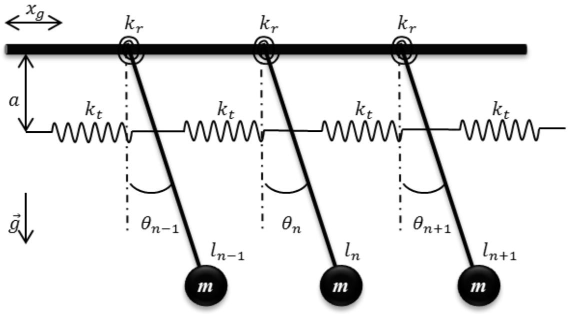

Figure 1. Periodic coupled pendulums chain.

The equation of motion of the n^{th} pendulum is written as follows:

$$ml_n^2\ddot{\theta}_n + cl_n^2\dot{\theta}_n + k_r\theta_n + k_t a^2\left(2\theta_n - \theta_{n-1} - \theta_{n+1}\right) = -ml_n\ddot{x}_g \tag{1}$$

To disturb the periodicity of the system, one pendulum is assumed to have slightly different length from the others. This mistuning is quantified by a length ratio α_n between the n^{th} pendulum length and the nominal length.

For simplification, dimensionless variables are defined as follows:

$$\theta_n = \Theta_n e^{j\omega t}; x_g = X_g e^{j\omega t}; \alpha_n = \frac{l_n}{l}; \omega_0 = \sqrt{\frac{k_r}{ml^2}}; \eta = \frac{c}{m\omega_0}; \beta = \frac{k_t a^2}{k_r}; f = \frac{X_g}{l}; \Omega = \frac{\omega}{\omega_0} \tag{2}$$

where η is the damping factor, β the coupling factor and $j^2 = -1$. Eqs. (1) and (2) lead to:

$$\left(-\alpha_n{}^2\Omega^2 + j\alpha_n{}^2\eta\Omega + 1\right)\theta_n + \beta\left(2\theta_n - \theta_{n-1} - \theta_{n+1}\right) = \alpha_n\Omega^2 f \quad n = 1...N \tag{3}$$

This system of equations is solved for each angular frequency of excitation Ω .

3 NUMERICAL RESULTS

Let's consider a chain of four weakly coupled pendulums with $\eta = 0.01$, $\beta = 0.005$, $f = 1$.

If the pendulum chain is perfectly periodic, the dimensionless eigenfrequencies are Ω_1=1.001, Ω_2=1.003, Ω_3=1.007, Ω_4=1.009 and conformity occurs between the amplitude pairs (Θ_1, Θ_4) and (Θ_2, Θ_3) reflecting the symmetry of the chain. This symmetry is broken when $\alpha_2 = 1.01$ ($\alpha_1 = \alpha_3 = \alpha_4 = 1$), as shown in figure 2.a. The dimensionless eigenfrequencies become Ω_1=0.994, Ω_2=1.003, Ω_3=1.006, Ω_4=1.008 and an amplitude mistuning occurs. The amplitude of the 2nd pendulum response is the highest with Θ_{2max}= 128.18.

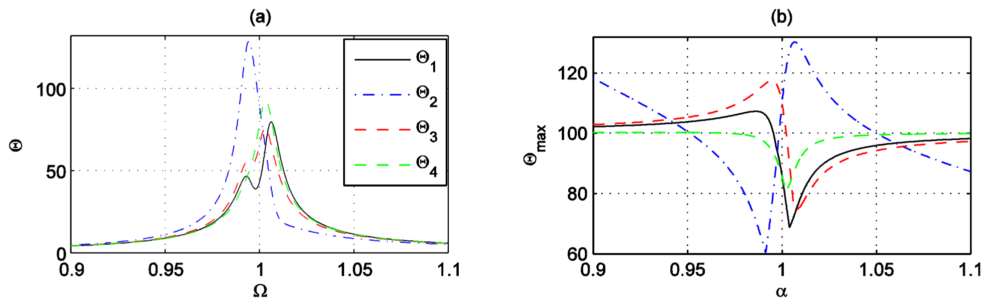

Figure 2. a. Dimensionless amplitudes for $\alpha_2 = 1.01$, b. Variation of maximal dimensionless amplitudes due to variation of α_2 from 0.9 to 1.1.

Small variation of α_2 from 0.9 to 1.1 causes significant variation of maximal amplitudes as shown in Figure 2.b. The difference $\Delta\Theta_{max}$ between higher maximal amplitude and lower one is highest at $\alpha_2 = 1.005$ ($\Delta\Theta_{max}$ = 59.09) between Θ_{2max} and Θ_{1max}. The symmetry between Θ_2 and Θ_3 is more disturbed than the symmetry between Θ_1 and Θ_4.

For more realistic representation of imperfection, we suppose that α_2 is an uncertain parameter which varies according to:

$$\alpha_2 = \alpha_0 \left(1 + \delta\xi\right) \tag{4}$$

where ξ is a Gaussian random variable, $\alpha_0 = 1$ and δ is the dispersion value.

The Latin Hypercube Sampling method is used with 1000 samples. The analysis of the trends in the output data (eigenfrequencies and amplitudes) is achieved by statistical evaluations: envelope (extreme statistics), dispersion (standard deviation / mean), skewness γ (distribution asymmetry) and kurtosis κ (heaviness of tail of the distribution).

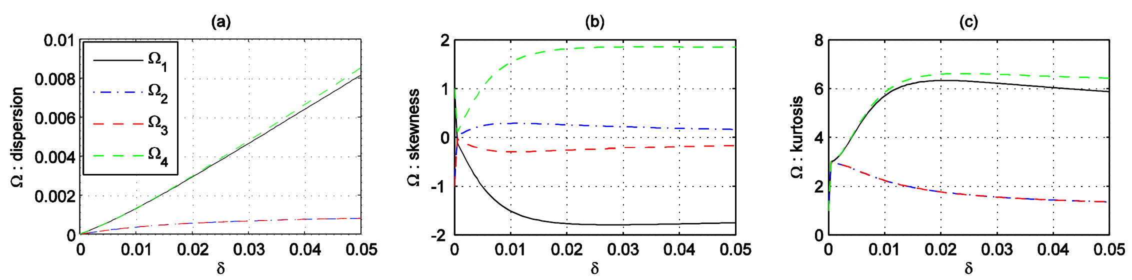

Figure 3. a. Dispersion, b. skewness and c. kurtosis of the stochastic dimensionless eigenfrequencies for $0 \le \delta \le 0.05$.

Figure 3 shows that the variation of Ω_1 and Ω_4 is much more important than the variation of Ω_2 and Ω_3. This is illustrated through increasing dispersions (Figure 3.a). Ω_2 and Ω_3 distributions are

fairly symmetrical (-0.5 < γ < -0.5, Figure 3.b). Nevertheless, Ω_1 and Ω_4 are highly skewed with asymmetrical distributions (γ < -1 or γ > 1) which are heavier than those of Ω_2 and Ω_3 (higher kurtosis values for Ω_1 and Ω_4, Figure 3.c).
The evolution of maximal amplitude with respect to δ is illustrated in figure 4. Higher dispersion is obtained for Θ_{2max} as shown in Figure 4.a. Smaller and nearly similar dispersions are obtained for Θ_{1max} and Θ_{3max} since 1st and 3rd pendulums are coupled to the disturbed one. Smallest dispersion is obtained for Θ_{4max} since the 4th pendulum is not directly coupled to the 2nd one. Maximal vibration localization is achieved for δ_m = 2.45% (Θ_{2max}=130.30) and remains constant up to δ = 5%. At δ = 3.3%, dispersion of Θ_2 reaches it maximum (24.44%) and decreases beyond. Up to δ = 3.3%, the Θ_{3max} distribution has heaviest (highest κ, Figure 4.c) long tail to the left (γ < 0, Figure 4.b), meaning that Θ_{3max} has the most tendency to decrease.

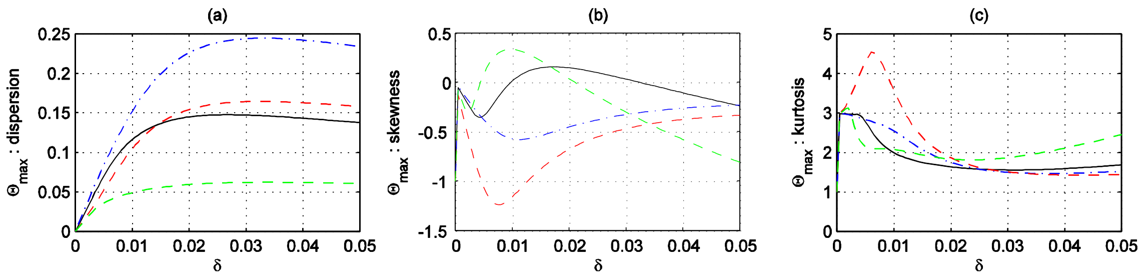

Figure 4. a. Dispersion, b. skewness and c. kurtosis of the stochastic maximal dimensionless amplitudes for $0 \le \delta \le 0.05$.

4 CONCLUDING REMARKS

Stochastic analysis of uncertain mistuning effects on vibration localization in near-periodic coupled pendulums chain was performed in this paper. Vibration localization reaches its maximum for a given dispersion level. Future work will consist in generalizing the proposed concept to Mdof near-periodic structures in order to extract the benefits of random imperfections in term of vibration localization. This denotes an interesting challenge for energy harvesting in presence of uncertainty, meriting particular attention.

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