STOCHASTIC ANALYSIS OF NEAR-PERIODIC COUPLED PENDULUMS CHAIN

K. Chikhaoui\textsuperscript{1}, N. Bouhaddi\textsuperscript{1}, N. Kacem\textsuperscript{1} and M. Ichchou\textsuperscript{2}

\textsuperscript{1}Univ. Bourgogne Franche-Comté, FEMTO-ST Institute, CNRS/UFC/ENSMM/UTBM, Department of Applied Mechanics, 25000 Besançon, France
\textsuperscript{2}LTDS, UMR 5513, Ecole Centrale de Lyon, Ecully, France
Email: khaoula.chikhaoui@femto-st.fr, noureddine.bouhaddi@femto-st.fr, najib.kacem@femto-st.fr, mohamed.ichchou@ec-lyon.fr

ABSTRACT

It is known that, when the mechanical coupling between the components is weak, small imperfections in a periodic structure can induce vibration localization. Stochastic analysis of near-periodic coupled pendulums chain is discussed in this paper. Perfect periodicity of the system is disturbed by varying randomly the length of one of the pendulums which is considered as an uncertain parameter. Its randomness is modeled in a probabilistic framework by a random variable according to a given range of dispersion level. Stochastic effects on vibration localization in mistuned four coupled pendulums chain is investigated through statistical evaluations. To do so, the propagation of uncertainties is performed using the Latin Hypercube Sampling method.
1 INTRODUCTION

Mistuning, or disorder, resulting from material defects, structural damage, manufacturing defaults, etc., breaks the perfect arrangement of periodic structures and alters significantly their dynamic behavior. The structure then becomes nearly periodic or called mistuned and vibration localization could occur under certain circumstances \[1\]. Zhu et al. \[2\] studied localization in randomly disordered coupled beams and proved that the wave propagation and localization can be altered by properly adjusting the structural parameters. Recently, Malaji et al. \[3\] investigated the effect of mistuning on vibration localization in two coupled pendulums chain. The main purpose of the present study is to investigate the stochastic effects of uncertain mistuning on vibration localization in a coupled pendulums chain.

2 MODEL

The scheme of \(N\) coupled pendulums chain is illustrated in figure 1. The pendulums have same mass \(m\), torsional stiffness \(k_r\) and proportional damping constant \(c\) and are weakly coupled by translational springs \(k_t\). An external base excitation \(x_g\) is applied to the system.

![Figure 1. Periodic coupled pendulums chain.](image)

The equation of motion of the \(n\)th pendulum is written as follows:

\[
ml_n^2 \ddot{\theta}_n + cl_n \dot{\theta}_n + k_r \theta_n + k_r a^2 \left( 2\theta_n - \theta_{n-1} - \theta_{n+1} \right) = -ml_n \ddot{x}_g \tag{1}
\]

To disturb the periodicity of the system, one pendulum is assumed to have slightly different length from the others. This mistuning is quantified by a length ratio \(\alpha\) between the \(n\)th pendulum length and the nominal length.

For simplification, dimensionless variables are defined as follows:

\[
\theta_n = \Theta_n e^{j\omega t} ; x_g = X_g e^{j\omega t} ; \alpha_n = \frac{l_n}{l} ; \omega_0 = \sqrt{\frac{k_r}{ml^2}} ; \eta = \frac{c}{m\omega_0} ; \beta = \frac{k_r a^2}{k_r} ; f = \frac{X_g}{l} ; \Omega = \frac{\omega}{\omega_0} \tag{2}
\]

where \(\eta\) is the damping factor, \(\beta\) the coupling factor and \(j^2 = -1\). Eqs. (1) and (2) lead to:

\[
\left( -\alpha_n^2 \Omega^2 + j\alpha_n^2 \eta \Omega + 1 \right) \dot{\theta}_n + \beta \left( 2\theta_n - \theta_{n-1} - \theta_{n+1} \right) = \alpha_n \Omega^2 f \quad n = 1...N \tag{3}
\]

This system of equations is solved for each angular frequency of excitation \(\Omega\).

3 NUMERICAL RESULTS

Let’s consider a chain of four weakly coupled pendulums with \(\eta = 0.01\), \(\beta = 0.005\), \(f = 1\).
If the pendulum chain is perfectly periodic, the dimensionless eigenfrequencies are \( \Omega_1=1.001, \Omega_2=1.003, \Omega_3=1.007, \Omega_4=1.009 \) and conformity occurs between the amplitude pairs \((\Theta_1, \Theta_4)\) and \((\Theta_2, \Theta_3)\) reflecting the symmetry of the chain. This symmetry is broken when \( \alpha_2 = 1.01 \) \((\alpha_1 = \alpha_3 = \alpha_4 = 1)\), as shown in figure 2.a. The dimensionless eigenfrequencies become \( \Omega_1=0.994, \Omega_2=1.003, \Omega_3=1.006, \Omega_4=1.008 \) and an amplitude mistuning occurs. The amplitude of the 2\(^{nd}\) pendulum response is the highest with \( \Theta_{2\text{max}} = 128.18 \).

Small variation of \( \alpha_2 \) from 0.9 to 1.1 causes significant variation of maximal amplitudes as shown in Figure 2.b. The difference \( \Delta \Theta_{\text{max}} \) between higher maximal amplitude and lower one is highest at \( \alpha_2 = 1.005 \) \((\Delta \Theta_{\text{max}} = 59.09)\) between \( \Theta_{2\text{max}} \) and \( \Theta_{1\text{max}} \). The symmetry between \( \Theta_2 \) and \( \Theta_3 \) is more disturbed than the symmetry between \( \Theta_1 \) and \( \Theta_4 \).

For more realistic representation of imperfection, we suppose that \( \alpha_2 \) is an uncertain parameter which varies according to:

\[
\alpha_2 = \alpha_0 (1 + \delta \xi)
\]

where \( \xi \) is a Gaussian random variable, \( \alpha_0 = 1 \) and \( \delta \) is the dispersion value.

The Latin Hypercube Sampling method is used with 1000 samples. The analysis of the trends in the output data (eigenfrequencies and amplitudes) is achieved by statistical evaluations: envelope (extreme statistics), dispersion (standard deviation / mean), skewness \( \gamma \) (distribution asymmetry) and kurtosis \( \kappa \) (heaviness of tail of the distribution).

Figure 3 shows that the variation of \( \Omega_1 \) and \( \Omega_4 \) is much more important than the variation of \( \Omega_2 \) and \( \Omega_3 \). This is illustrated through increasing dispersions (Figure 3.a). \( \Omega_2 \) and \( \Omega_1 \) distributions are
fairly symmetrical (-0.5 < γ < -0.5, Figure 3.b). Nevertheless, Ω₁ and Ω₄ are highly skewed with asymmetrical distributions (γ < -1 or γ > 1) which are heavier than those of Ω₂ and Ω₃ (higher kurtosis values for Ω₁ and Ω₄, Figure 3.c).

The evolution of maximal amplitude with respect to δ is illustrated in Figure 4. Higher dispersion is obtained for Θ₂max as shown in Figure 4.a. Smaller and nearly similar dispersions are obtained for Θ₁max and Θ₃max since 1st and 3rd pendulums are coupled to the disturbed one. Smallest dispersion is obtained for Θ₄max since the 4th pendulum is not directly coupled to the 2nd one. Maximal vibration localization is achieved for δₘ = 2.45% (Θ₂max=130.30) and remains constant up to δ = 5%. At δ = 3.3%, dispersion of Θ₂ reaches its maximum (24.44%) and decreases beyond. Up to δ = 3.3%, the Θ₃max distribution has heaviest (highest κ, Figure 4.c) long tail to the left (γ < 0, Figure 4.b), meaning that Θ₃max has the most tendency to decrease.

Figure 4. a. Dispersion, b. skewness and c. kurtosis of the stochastic maximal dimensionless amplitudes for 0 ≤ δ ≤ 0.05.

4 CONCLUDING REMARKS

Stochastic analysis of uncertain mistuning effects on vibration localization in near-periodic coupled pendulums chain was performed in this paper. Vibration localization reaches its maximum for a given dispersion level. Future work will consist in generalizing the proposed concept to M dof near-periodic structures in order to extract the benefits of random imperfections in term of vibration localization. This denotes an interesting challenge for energy harvesting in presence of uncertainty, meriting particular attention.

ACKNOWLEDGMENT

This project has been performed in cooperation with the Labex ACTION program (project Recup’Aimant 2017).

REFERENCES

