

VIBRO-ACOUSTIC ENERGY FLOW ON A CAR FLOOR STRUCTURE USING DYNAMICAL ENERGY ANALYSIS

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ABSTRACT

Dynamical Energy Analysis (DEA) in the form of Discrete Flow Mapping (DFM) is a fairly new mesh-based method for numerically modelling structure borne sound transmission in complex structures. A key feature is the possibility to work directly on existing finite element (FE) meshes avoiding time-consuming and costly re-modelling. Furthermore, DFM provides detailed spatial information about the vibrational energy distribution within a complex structure in the mid-to-high frequency range. In this work we will illustrate the method using a car floor structure which consists of a big panel and several rails connected by spot welds modeled in FE through Rigid Body Elements (RBE).

1 INTRODUCTION

Simulations of the vibro-acoustic properties of complex structures (such as cars, ships, airplanes, etc.) are routinely carried out in various design stages. For low frequencies, the established method of choice is the finite element method (FEM). But high frequency analysis using FEM requires extremely fine meshes of the body structure to capture the shorter wavelengths and is therefore computationally very costly. Furthermore the structural response at high frequencies is very sensitive to small variations in material properties, boundary conditions etc. This makes the output of a single FEM calculation less reliable and makes ensemble averages necessary furthermore enhancing computational cost. Therefore at high frequencies other numerical methods with better computational efficiency are preferable.

The Statistical Energy Analysis (SEA) [1] has been developed to deal with high frequency problems and leads to relatively small and simple models. However, SEA is based on a set of often hard to verify assumptions, which effectively require diffuse wave fields and quasi-equilibrium of wave energy within (weakly coupled and weakly damped) sub-systems.

One alternative to SEA is to instead consider the original vibrational wave problem in the high frequency limit, leading to a ray tracing model of the structural vibrations. The tracking of individual rays across multiple reflection is not computational feasible because of the proliferation of trajectories. Instead, a better approach is tracking densities of rays propagated by a transfer operator. This forms the basis of the Dynamical Energy Analysis (DEA) method introduced in [2]. DEA can be seen as an improvement over SEA where one lifts the diffusive field and the well separated subsystem assumption. One uses an energy density which depends both on position and momentum. DEA can work with relatively fine meshes where energy can flow freely between neighboring mesh cells. No remodeling as for SEA is necessary as DEA can use meshes created for a FE analysis. Also finer structural details than SEA can be resolved.

In this paper, we apply the DEA method to a caravan car floor structure. The floor structure consists of a floor panel, two longitudinal rails and six transverse rails, all built up from 2D plate elements, as illustrated in Fig. 1. The floor panel is connected to the rails through a number of spot-welds. First we discuss details of DEA itself and then we we present numerical results comparing DEA with FEM calculations.



Figure 1: (a) car floor and (b) spotweld. The spotweld is modelled through RBEs (red) and a small solid element (yellow) in the middle.

2 DYNAMICAL ENERGY ANALYSIS / DISCRETE FLOW MAPPING

The implementation of DEA on meshes is called Discrete Flow Mapping (DFM). We will here briefly describe the idea behind DFM, for details see [3]. In DFM it is possible to compute vibroacoustic energy densities in complex structures at high frequencies, including multi-modal propagation and curved surfaces. DFM is a mesh based technique where a transfer operator is used to describe the flow of energy through boundaries of subsystems of the structure; the energy flow is represented in terms of a density of rays ρ , that is, the energy flux through a given surface is given through the density of rays passing through the surface at point *s* with direction p. Here, s parametrises the surface and p is the direction component tangential to the surface. In what follows, the surfaces is represented by the union of all boundaries of the mesh cells of the FE mesh describing the car floor. The density $\rho(s, p) = \rho(X_s)$, with phase space coordinate $X_s = (s, p)$, is transported from one boundary to the next boundary intersection via the boundary integral operator [3]

$$\mathcal{B}[\rho](X'_s) := \int w(X'_s)\delta(X'_s - \phi(X_s))\rho(X_s) \,\mathrm{d}X_s \tag{1}$$

where $\phi(X_s)$ is the map determining where a ray starting on a boundary segment at point s with direction p_s passes through another boundary segment, and $w(X_s)$ is a factor containing damping and reflection/transmission coefficients (akin to the coupling loss factors in SEA). It also governs the mode conversion probabilities in the case of both in-plane and flexural waves, which are derived from wave scattering theory [4].

In a next step, the transfer operator (1) is discretised using a set of basis functions of the phase space. Once the matrix B has been constructed, the final energy density ρ on the boundary phase-space of each element is given in terms of the initial density ρ_0 by the solution of a linear system of the form

$$(\mathbf{1} - \mathbf{B})\rho = \rho_0. \tag{2}$$

The integral in (1) can be adapted to incorporate further complexity and refinement in a DFM model. The vehicle floor in Fig. 1 contains spot welds fixing the stiff rails to the floor panel. This is modelled in the FE model with the connections shown in Fig. 1b, here in terms of a set of RBEs (red lines) together with solid element modelling extra mass and stiffness of the spot weld. The RBEs describe here constraint conditions and make it possible to transfer forces directly from one mesh to another. Such a set-up can not be used in a DFM treatment which is based on modelling energy flow through surfaces and mesh boundaries.

In order to avoid costly remodelling of the structure, in DFM we describe the energy transfer across spot weld by introducing coupling elements between edges connected to the spot welds both in the 'upper' and 'lower' sheet. Energy arriving at an edge connected to a spot weld is distributed uniformly (also in direction) among all neighbouring edges.

3 NUMERICAL RESULTS

In order to compare the different numerical approaches, first we have calculated the spatial kinetic energy distribution originating from a single (perpendicular) point excitation on the plain floor panel without rails (shown as component 9 in Fig. 1a). The DFM results are compared to one-third octave band frequency-averaged FEM results, with the band average at 2500 Hz. Note that the DFM calculation uses only the band average frequency. The calculation uses a hysteretic damping loss factor of $\eta = 0.04$.

The results are shown in Fig. 2a. The energy distribution predicted by FEM and DFM is very non-uniform and would not be well-captured by an SEA model. In contrast to SEA, DFM gives also the spatial distribution information, which is in close agreement with the FEM results. In particular, we see the directional dependence of the energy flow, which is predominantly in the horizontal direction as plotted. This is caused by several horizontally extended out-of-plane bulges. It is only in the lower right part of the panel, with negligible energy content, that deviations between the FEM and DFM predictions are visible. The results also show a good quantitative agreement. In particular, the total kinetic energy given by the DFM prediction is within 12% of the FEM prediction.

In a next step, we calculate the response of the full car floor model shown in Fig. 1a. This includes the coupling of the rails to the floor panel and between different rails via the spot-weld



Figure 2: The kinetic energy distribution on a logarithmic color scale ((b) shifted by 5 dB relatively to (a)). The (frequency averaged) FEM results (upper panels) are compared to DEA results (lower panels). The left panels (a) show the bare floor panel, the right panels (b) show the full structure including rails.

models depicted in Fig. 1b. The point loading is now applied on top of a rail, but otherwise the scenario is equivalent to the previous calculation. The results are shown in Fig. 2b. The deviations between the FE result and the DEA result are within 18% when integrated over the total area of the car floor. A detailed analysis shows, that the energy is less pronounced in the DEA calculation compared to the FE calculation when moving away from the source. This suggest that the modelling of the coupling between different components is currently too weak in the DEA model, which calls for a more refined DEA modelling of the RBE connections.

4 CONCLUDING REMARKS

We have tested the DFM method for a car-floor structure at mid- and high- frequencies. The combination of the thin shell floor panel connected to a number of stiffer rails via spot welds poses challenges for a DFM calculations. We have developed a method for treating the coupling of different FE meshes via RBEs in a DFM simulation. The results compare well with (frequency-band averaged) FE calculations both for the floor panel alone and for the full car floor structure. Improvements of the coupling in the DEA set-up needs to be considered.

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