DYNAMICAL REGIMES FOR A TIME-CORRELATED RANDOMLY EXCITED BOUNCING BALL MODEL

J. Perret-Liaudet[1], C. Zouabi[2] and J. Scheibert[1]

[1]Laboratoire de Tribologie et Dynamique des Systèmes, UMR 5513
École centrale de Lyon, member of the Université de Lyon, F69134 Écully, FRANCE
Email: joel.perret-liaudet@ec-lyon.fr, julien.scheibert@ec-lyon.fr

[2]CESI - Lyon, 19 Avenue Guy de Collongue - F69130 Écully, FRANCE
Email: czouabi@cesi.fr

ABSTRACT

The popular bouncing ball model, which consists in a ball submitted to the gravitational field and bouncing vertically on a vibrating plate with inelastic impacts, is under study in this paper. Contrary of most of studies witch assume a harmonic vertical motion of the plate, one considers random excitations of the ball induced by the plate motion. More precisely, we consider the dynamic behaviour of a revisited stochastic version of the bouncing ball model, by introducing the table displacement as a continuous time Gaussian random process with tunable correlation time. The memory effect of the excitation is then analysed, by investigating the dynamic behaviour through numerous numerical simulations. One shows that the dynamic behaviour is not only governed by the restitution coefficient at impacts and the dimensionless excitation amplitude level, but also by the correlation time of the excitation process. One highlights the relevant parameter that distinguishes the well-differentiated dynamic regimes. Quickly says, this dimensionless parameter depends for the essential from the bandwidth of the excitation signal.

1 INTRODUCTION

The popular bouncing ball (BB) model, which consists in a ball submitted to the gravitational field and bouncing vertically on a vibrating plate with inelastic impacts, has been widely studied in the last decades. This is due to both its simplicity and the amazing richness of its dynamics, from harmonic to chaotic, through subharmonic and quasi-periodic responses. It is now one of the paradigms for nonlinear dynamics and chaos (see, *e.g.* [1, 2] for BB in textbooks). Most of the studies achieved to date consider harmonic vertical motion of the plate. On the contrary, few of them include random vibrations of the plate, in spite of its undeniable relevance. Moreover, the excitation induced by the plate motion at successive bounces is generally assumed to be a discrete Markovian memoryless stochastic process. However, the real plate motions are always characterized by a finite auto-correlation time t_{corr} below which the signal keeps memory of its previous values. The Markovian assumption of independent successive impacts corresponds to the fact that the ballistic flight time of the ball between two successive bounces is much larger than t_{corr}. This is the case with the so-called Chirikov conditions [3]. Conversely, for two bounces separated by a short flight time, the two relevant plate velocities can be strongly correlated. Thus, in regimes in which short flight times are dominant, the standard Markovian approach is expected to fail to capture the BB model dynamics.

In this context, the main purpose of this study is to characterize the BB model dynamics with stochastic excitation, when memory effects cannot be neglected.

2 THE REVISITED BOUNCING BALL MODEL

Consider the popular BB model (see Figure 1) consisting on a point-like ball of finite mass bouncing vertically under the action of gravity, g, on an infinitely massive vibrating plate, the originality of our model is to introduce, for the vibrating plate, a correlated stochastic motion, $h(t)$, with tunable correlation time, t_{corr}. To this end, $h(t)$ is obtained from an uncorrelated Gaussian white noise $\psi(t)$, filtered by a second-order filter as

$$\ddot{h} + 2\zeta\Omega\dot{h} + \Omega^2 h = \psi(t) \tag{1}$$

with Ω being the center frequency of the filter and ζ its damping coefficient. Note that ζ is related to the frequency contents of the signal because the bandwidth of its power spectrum density (PSD), $S_{hh}(\omega)$ is equal to $2\zeta\Omega$. The autocorrelation function $< h(t)h(t+\tau) >$ of h is equal to $\sigma_h^2 \exp(-\zeta\Omega|\tau|)\, f(\tau)$ with f a periodic function and σ_h the standard deviation of h, so the correlation time can be defined as $t_{corr} = 1/\zeta\Omega$. To avoid infinite energy in the acceleration signal the displacement is further filtered by a first-order low-pass filter with a cutoff frequency higher than Ω. Typical simulated PSDs are shown in Figure 2 for the two cases, narrow and broadband.

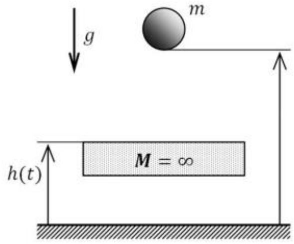

Figure 1. Sketch of the bouncing ball (BB) model.

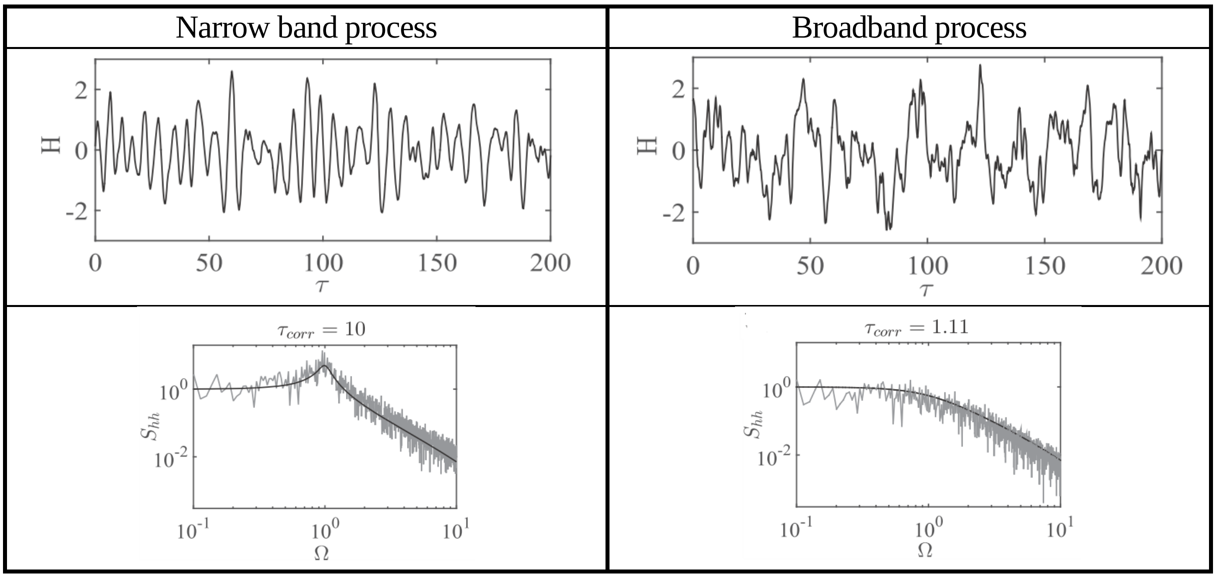

Figure 2. Typical realizations of the dimensionless plate time-displacement and its power spectral densities (PSD): narrow and broadband cases.

Now, for any generated excitation signal, we then solve the bouncing ball problem by calculating the values of the post impact velocity, v_n, and instant, t_n, of the all-successive impacts. In practice, we solve the following equations:

$$t_{n+1} = t_n + \theta_n \tag{2}$$

$$v_{n+1} = -e(v_n - g\theta_n) + (1+e)w_{n+1} \tag{3}$$

with e the restitution coefficient introduced to take into account the partially inelastic impact characteristic, w_n the plate velocity at impact, and θ_n the flight time obtained from the following equation:

$$-\frac{1}{2}g\theta_n^2 + v_n\theta_n + h_n - h_{n+1} = 0 \tag{4}$$

In fact, Equations (2) and (3) define the classical Poincaré map for the BB model. A dimensional analysis shows that the system is governed only by three dimensionless parameters, i.e. the restitution coefficient e, the reduced plate acceleration defined by $\Lambda = \sigma_w^2/g\sigma_h$ and the dimensionless correlation time $\tau_{corr} = \Omega t_{corr}$. On this basis, we have performed simulations for a large number of values of this triplet [4].

3 RESULTS

Typical probability density functions (pdf) of the dimensionless take-off velocity $V_n = v_n/\sigma_w$ are shown in Figure 3. Wood and Byrne have studied the case of a completely uncorrelated Markovian excitation [5], and their results (velocity quoted V_{WB}) are used as a reference to highlight the differences brought by our improved model which includes the correlation in the excitation. As we can see, memory effects become negligible when individual flight times are larger than the correlation time. This case is favoured for large excitations Λ and/or short correlation time τ_{corr}.

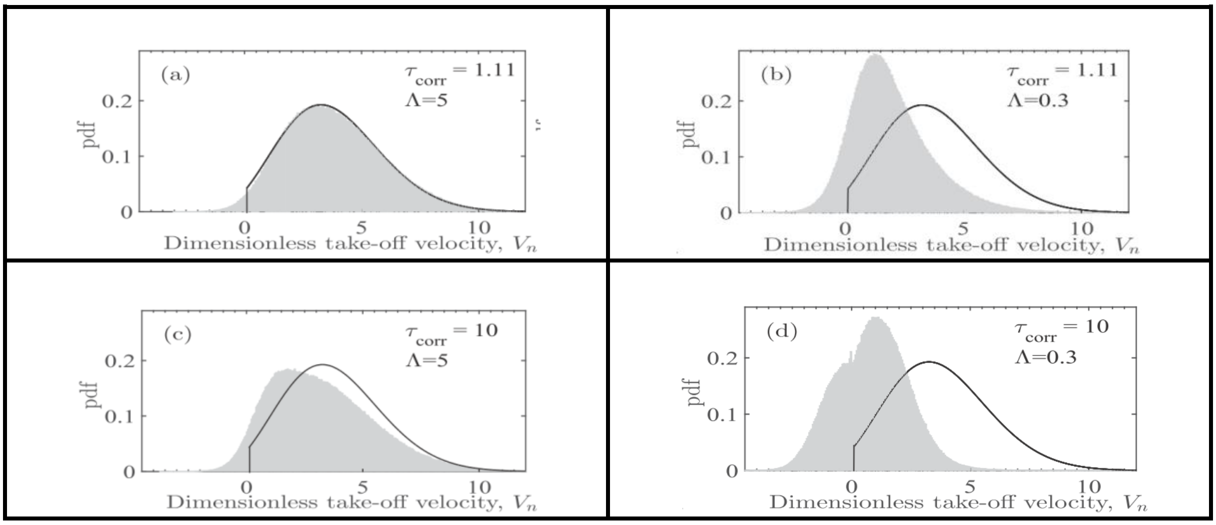

Figure 3. Typical pdf of the dimensionless take-off velocity for selected Λ and τ_{corr}, with the example of $e = 0.8$. Solid lines correspond to the Wood and Byrne results.

A detailed analysis [6] shows that the relevant parameter consists on the ratio of the Markovian mean flight time of the ball and the mean time between successive peaks in the plate motion. This dimensionless parameter depends on the bandwidth of the excitation signal. When the parameter is large, the Markovian approach is appropriate; but for low levels, memory effects become not negligible leading to a significant decrease of jump durations; and finally at smallest values of the ratio, chattering occurs.

ACKNOWLEDGMENT

This work was supported by the LABEX MANUTECH-SISE (ANR-10-LABX-0075) of Université de Lyon, within the program "Investissements d'avenir" (ANR-11-IDEX-0007) operated by the French National Research Agency (ANR). The research leading to these results has received funding from the People Programme (Marie Curie Actions) of the European Union's Seventh Framework Programme (FP7/2007-2013) under REA grant agreement no. PCIG-GA-2011-303871.

REFERENCES

[1] N. Tufillaro, and T. Abbot, and J. Reilly. *An experimental approach to nonlinear dynamics and chaos*. Addison-Wesley, Redwood City, 1992.

[2] J. Guckenheimer, and P. Holmes. *Nonlinear oscillations, dynamical systems, and bifurcations of vector fields*. Springer-Verlag, New-York, 1993.

[3] B. Chirikov. A universal instability of many-dimensional oscillator systems. *Physics reports*. 52(5):263-379, 1979.

[4] C. Zouabi. *Dynamique d'un contact glissant rugueux-rugueux sous faible charge: experiences et modélisation*. Thèse de doctorat, École centrale de Lyon, 2016.

[5] L. Wood, and K. Byrne. Analysis of a random repeated impact process. *J. Sound and Vibration*. 78: 329-345, 1981.

[6] C. Zouabi, J. Scheibert, and J. Perret-Liaudet. Memory in random bouncing ball dynamics. *EPL*. 115(50006) 5p., 2016.