MODAL INTERACTIONS IN A TWO-NANOMECHANICAL-RESONATOR ARRAY

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ABSTRACT

Most studies on nanomechanical resonators in the literature are concerned with a single resonator. In this work, an array of two nanomechanical resonators is analyzed. A quasi-analytic approach with averaging method is used to compare the beams responses with and without electrostatic coupling terms. The results show modal interactions between the two beams due to the electrostatic coupling. It is shown that the qualitative behavior of the coupled resonators can be infered from the response curves of the uncoupled resonators. In particular, additional loops occur due to the algebraic structure of the coupled system. The contribution lies in the deduction of the beam array responses curve by using multiple uncoupled responses of the single-beam resonators.

1 INTRODUCTION

Arrays of MEMS or NEMS present complex dynamical behaviors due to electric, magnetic and mechanic nonlinear couplings. Lifshitz and Cross [1] studied the responses of n electrical and mechanical coupled oscillators with parametric resonance in the low nonlinear limit by using a perturbation method. Gutschmidt and Goettlieb worked on arrays with electrical coupling. They focused on the n-beam dynamic behaviour in the region of internal one-to-one, parametric and several internal three-to-one resonances corresponding to low, medium and large DC voltages [2]. Kacem et al. developped a single beam model to investigate the sensitivity of the resonance with respect to the electrostatic forcing. Their researches were carried out using averaging method validated by HBM+ANM [3]. In this paper, an array of $n = 2$ identical clamped-clamped beams is also considered but coupled only by an electrostatic force in order to study the modal interactions between the two beams due to the electrostatic coupling.

2 ARRAY OF TWO NANOMECHANICAL RESONATORS

A 2-moving-beam array is considered, as sketched in Figure 1. The two beams located at the ends of the array are fixed and serve only as electrostatic actuator. All 4 beams are identical. l, b, h, I, g are the dimensions of the beams, i.e., length, width, height, moment of inertia, gap between two adjacent beams. E, ρ be the Young's modulus and the material density. Each

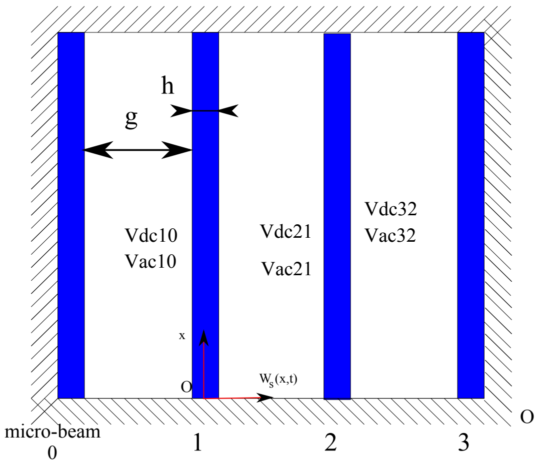


Figure 1: Array of two clamped-clamped M/NEMS beams.

beam is an electrostatic actuator for its adjacent beams. $V_{s,s+1} = V_{dcs,s+1} + V_{acs,s+1}\cos(\Omega t)$ is the voltage applied between the successive beams s and $s+1$ with V_{dc}, V_{ac} the continuous and alternative voltages. The equation of the beam s in bending is as follows [2].

$$EI\frac{\partial^4 \tilde{w}_s(\tilde{x},\tilde{t})}{\partial \tilde{x}^4} + \rho bh\frac{\partial^2 \tilde{w}_s(\tilde{x},\tilde{t})}{\partial \tilde{t}^2} - \left[\tilde{N} + \frac{Ebh}{2l}\int_0^l \left(\frac{\partial \tilde{w}_s(\tilde{x},\tilde{t})}{\partial \tilde{x}}\right)^2 \mathrm{d}\tilde{x}\right]\frac{\partial^2 \tilde{w}_s(\tilde{x},\tilde{t})}{\partial \tilde{x}^2}$$
$$= \frac{\epsilon_0 b C_n}{2}\left[\frac{V_{s,s+1}^2}{\left(g+\tilde{w}_{s+1}-\tilde{w}_s\right)^2} - \frac{V_{s-1,s}^2}{\left(g+\tilde{w}_s-\tilde{w}_{s-1}\right)^2}\right] \quad (1)$$

with $s = 1, 2$. Let ϵ_0, C_n be the dielectric constant and fringing the coefficient respectively. $\tilde{N}$ represents the lineic load along the x-axis. The beams 1 and 2 are clamped-clamped.

3 AVERAGING METHOD

The responses of the beams are more complicated than those of a single beam resonator. A quasi-analytic solution obtained by the averaging method can be used to explain why the electrostatic coupling generates additional loops onto the responses. First, the beam lateral deflection is expanded on its fundamental mode only:

$$w_1(x,t) = \phi_1(x)a_{11}(t), \qquad w_2(x,t) = \phi_1(x)a_{21}(t) \tag{2}$$

First-order Taylor series are then used to simplify the analytic calculation:

$$\frac{1}{(1+w_{s+1}-w_s)^2} \simeq 1 - 2(w_{s+1}-w_s), \qquad \frac{1}{(1+w_s-w_{s-1})^2} \simeq 1 - 2(w_s - w_{s-1}) \tag{3}$$

Since the resonators of the 2-beam array have the same boundary conditions, their eigenmodes are identical. Therefore, a Galerkin method is used to eliminate the spatial dependence from the equation of motion (1). Then using the averaging method and considering the solutions $a_{11}(t), a_{21}(t)$ in following forms

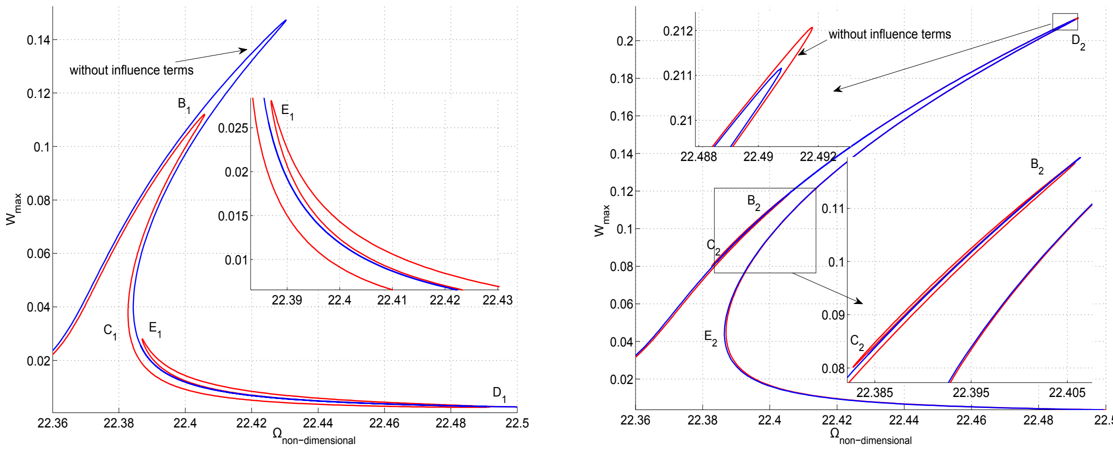
$$a_{11} = A_{11}(t)\cos(\Omega t) + B_{11}(t)\sin(\Omega t), \tag{4}$$
$$a_{21} = A_{21}(t)\cos(\Omega t) + B_{21}(t)\sin(\Omega t), \tag{5}$$

yield

$$\begin{aligned}
\Omega\dot{A}_{11} = \quad & -B_{11}\omega_1\epsilon\sigma_1 - \frac{1}{2}cA_{11}\Omega + \frac{3}{8}\beta_{11}A_{11}^2B_{11} + \frac{3}{8}\beta_{11}B_{11}^3 \\
& +\frac{1}{8}\beta_{13}B_{11} + \frac{1}{2}\delta_{11}B_{21} + \frac{1}{8}\delta_{13}B_{21} && (6)\\
\Omega\dot{B}_{11} = \quad & \frac{1}{2}cB_{11}\Omega + \frac{3}{8}\beta_{11}A_{11}^3 + \frac{3}{8}\beta_{13}A_{11} + \frac{1}{2}\gamma_{12} + \frac{3}{8}\beta_{11}A_{11}B_{11}^2 \\
& -A_{11}\omega_1\epsilon\sigma_1 + \frac{1}{2}\delta_{11}A_{21} + \frac{3}{8}\delta_{13}A_{21} && (7)\\
\Omega\dot{A}_{21} = \quad & -B_{21}\omega_2\epsilon\sigma_2 - \frac{1}{2}cA_{21}\Omega + \frac{3}{8}\beta_{21}A_{21}^2B_{21} + \frac{3}{8}\beta_{21}B_{21}^3 \\
& +\frac{1}{8}\beta_{23}B_{21} + \frac{1}{2}\delta_{21}B_{11} + \frac{1}{8}\delta_{23}B_{11} && (8)\\
\Omega\dot{B}_{21} = \quad & \frac{1}{2}cB_{21}\Omega + \frac{3}{8}\beta_{21}A_{21}^3 + \frac{3}{8}\beta_{23}A_{21} + \frac{1}{2}\gamma_{22} + \frac{3}{8}\beta_{21}A_{21}B_{21}^2 \\
& -A_{21}\omega_2\epsilon\sigma_2 + \frac{1}{2}\delta_{21}A_{11} + \frac{3}{8}\delta_{23}A_{11} && (9)
\end{aligned}$$

where the coefficients β_{ij}, γ_{ij}, δ_{ij} depend on the beam characteristics and on the applied voltages. They are not detailed here for the sake of conciseness. In Equations (6)-(9), the coupling terms $(\frac{1}{2}\delta_{11} + \frac{1}{8}\delta_{13})B_{21}$ and $(\frac{1}{2}\delta_{11} + \frac{1}{8}\delta_{13})A_{21}$ represent the influence of the second beam on the first beam and the coupling terms $(\frac{1}{2}\delta_{21} + \frac{1}{8}\delta_{23})B_{11}$, $(\frac{1}{2}\delta_{21} + \frac{3}{8}\delta_{23})A_{11}$ the influence of the first beam on the second beam. When $\dot{A}_{11} = \dot{B}_{11} = \dot{A}_{21} = \dot{B}_{21} = 0$ the steady-state motions appear. The corresponding nonlinear algebraic system is solved by an adapted numerical method, the obtained approximated solution is in agreement with a reference solution obtained by HBM+ANM [3] not shown here.

In order to analyze the influence of the coupling terms, Equations (6)-(9) with and without coupling terms are examined. The response curves with (red curves) and without (blue curves) coupling terms are plotted in Figure 2. Without these coupling terms Equations (6)-(7)



(a) (b)

Figure 2: Design 1 without added mass, Response by neglecting the coupling terms (blue), complete response (red). (a): first-beam response, (b): second-beam response.

and (8)-(9) form two independent systems of equations. Therefore, the responses of the two beams, (blue response curves in Figure 2) are similar to two single-beam responses.

With coupling terms Equations (6)-(9) are dependent and share the same bifurcation points and stability. When a bifurcation point is present on a response curve, the same bifurcation point will also be present at the same frequency on the other beam response. Therefore, the limit points originated from the responses without coupling terms will be present on all the other beam responses with coupling terms. This leads to the appearance of additional limit points on the response curves. In Figure 2, the limit points B_1 and C_1 on the first beam response generate at the same frequencies the loop $B_2 - C_2$ on the second-beam response. In the same way, D_2 and E_2 on the second-beam response produce the loop $D_1 - E_1$ on the first-beam response.

4 CONCLUSION

A quasi-analytic analysis with the averaging method of a two-nanomechanical-resonator array has been carried out. The existence of modal interactions between the two beams due to the electrostatic coupling has been enlightened. The appearance of additional loops onto response curves has been explained. The form of the response curves of an electrostatic coupled beam array can be anticipated using the uncoupled single-beam-resonator responses. This research represents an increment towards the comprehension and modeling of resonator arrays for application in mass sensing.

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