# VERTICAL DYNAMICS OF TWO SLIDING ROUGH SURFACES: COMPARISON BETWEEN NUMERICAL AND ANALYTICAL APPROACHES TO DESCRIBE THE EXCITATION SOURCE

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## ABSTRACT

*The vibration induced by the roughness of two surfaces sliding one upon each other is the excitation source for a so-called roughness noise. This source is a stochastic process, the spectral and statistic properties of which have to be described. Often the topographic properties of the surfaces are known. However, those properties are not related in a simple way to the excitation. The latter is determined by the statistics of the highest asperities of each surface. The aim of this study is to explore this relationship between topography and vibration. To achieve this goal, numerical and analytical methods have been implemented*

## 1 INTRODUCTION

When two rough surfaces are sliding one upon each other under weak load, the strains of the asperities are low and a macroscopic vertical vibration of the two solids appears. This vibration can be the source of jumps [1, 2] and is also a value of interest for the calculation of dispersive forces [3]. However this vibration can not be directly associated to the combination of the two surface topographies. Indeed only the highest asperities of each surface are in contact. The aim of this study is to characterize this geometrical filtering. To address this issue, the statistical properties of the contact of the highest asperities have to be understood. Analytical methods for extremal events exist and are commonly used in finance, dimensioning and quantification of risks [4]. To adapt such methods to our problem, a simplified system is studied considering the relative distance between two surfaces touching in only one contact point and sliding one upon each other. Let us call the upper one the slider which is considered square. An analytical calculation is done and compared to a numerical simulation of the problem. These simulation enable conclusions about the spectral properties of this vibration.

## 2 ANALYTICAL METHOD

Searching the first contact point between two surfaces is equivalent to find the maximum of a process that has the properties of the sum of the two surfaces. This problem of maxima has been widely studied. To apply those calculations to our case, a discrete way to describe surfaces with independent variables has been used. A surface is described through the height probability density function (pdf) and a power spectral density function (PSD). To convert this continuous representation to a convenient discrete description of the surfaces let us define the correlation length $\lambda$ of the surface. If two points of the surface are further than $\lambda$, they are completely uncorrelated. If $L$ is the side length of the slider, we can thus define a number of points that are independent and represent the surface, with $N = \frac{L^2}{\lambda^2}$. We assume that those $N$ points have the same pdf $p$ as the sum of the surfaces. Introducing the associated cumulative density function (cdf) $\mathcal{P}$ and following [4], the pdf $g$ of the height of the maximum $z$ is:

$$g(z) = Np(z)\mathcal{P}(z)^{N-1} \tag{1}$$

The two surfaces sliding one upon each other can be considered as a sequence of draws of a maximum. The vertical vibration statistics should then follow the law given in equation 1. Examples for various $N$ are given in the figure 1 considering $p$ Gaussian.

## 3 NUMERICAL METHOD

To compare to the results given by the analytical method presented above, the first contact point problem is now studied numerically. Realistic rough surfaces are generated numerically from typical PSD (rectangular shaped and fractal) with a Gaussian height distribution. The topography of a slider and a track, 5 times longer than the slider, are generated. A typical example of the topographies is shown on the figure 2. The slider is placed on one side of the track and move along it with a discretization step equal to the surface discretization. No rocking motion is taken into account. Let $x$ be the horizontal offset from the initial position. The first contact point when approaching the two surfaces is searched for each discretization point. All height differences between the two median planes are collected giving the first contact point profile $h(x)$. A typical histogram of $h$ is given in figure 3. Spectral properties can also be deduced from the numerous simulations.

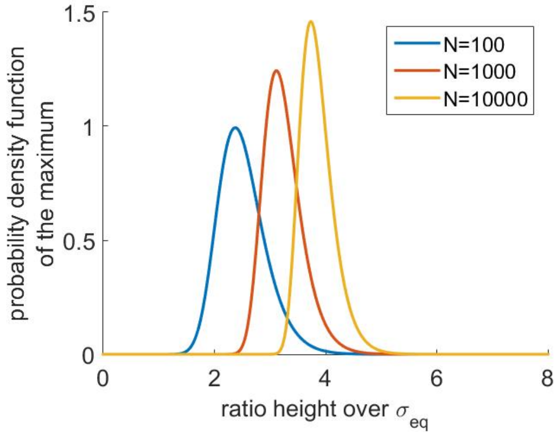


Figure 1: pdf of the maximum for various $N$. The height is normalized by $\sigma_{eq}$, the standard deviation of the sum of the surfaces

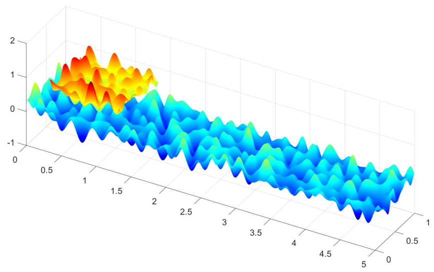

Figure 2: typical example of the topographies used to obtain the first contact point profile (arbitrary scale)

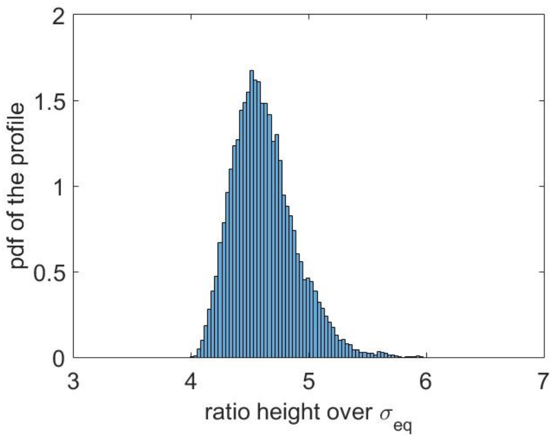


Figure 3. histogram of the numerical first contact point profile

## 4 COMPARISON AND CONCLUSION

The shapes of the pdf plotted in figures 1 and 3 are relatively similar. Numerous numerical results have been compared to an analytical model exhibiting a good agreement between the numerical and analytical results. Simulations have also brought spectral description of those signals. This work enables then a complete description of the vibration due to the roughness of two surfaces. It can then be used as an excitation source for dynamical models such as the bouncing ball [5].

## REFERENCES

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