GLOBAL SENSITIVITY ANALYSIS ON SANDWICH PANEL'S ACOUSTIC CHARACTERISTICS WITH CORRELATED INPUTS

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ABSTRACT

In this paper, a piece of sandwich composite material's mechanical parameters have been investigated to reveal the influence of its sound transmission properties using some Global Sensitivity Analysis (GSA) methods. Particularly, the correlation among these variables, which is caused by the core layer's meso-structure designs, is taken into consideration. For this purpose, some advanced Fourier Amplitude Sensitivity Test (FAST) algorithms are applied on the classic Mead's analytical sound transmission model. The test results show that the correlation and the variation of meso-structures both can greatly change the influence of these mechanical parameters on the characteristic sound transmission properties, the transmisson loss, for example. The statistics obtained in this research indicate that, in the industrial production process, the importance of some variables's uncertainty control should be re-evaluated when taking into consideration their distribution dependence caused by different meso-structures.

1 INTRODUCTION

In about recent 30 years, very fast development has been observed in the engineering research and industrial applications of sandwich composite panels. These panels have great advantage in their stiffness-to-mass ratio, leading to wide application in aeronautic and civil engineering, which raises the importance of evaluating and improving their sound proof capacity. Produced in high quantity and applied in various strict conditions, their quality control practices become more important than ever in the design and production phase. There are some researches that have been published with the implementation of some Global Sensitivity Analysis (GSA) methods, such as the one of Christen et al. [1]. But with the fast development of sandwich panels' meso-structures[2], the correlation effects can no longer be ignored.

2 SOUND TRANSMISSION IN SANDWICH COMPOSITE MATERIALS

Considering a piece of material's sound proof capacity, the transmission loss is a direct indicator of energy attenuation for a sound wave propagating through the material with a certain angle. For an analytical estimation, we prefer to use the classical Mead's model[3] with some corrections and simplifications by Clarkson and Ranky [4] . Under several assumptions and approximations, the vibration equation can be represented in this form:

$$D_f\nabla^6 w - g'(D_f + D)\nabla^4 w + m\omega^2\nabla^2 w - mg'\omega^2 w = \nabla^2 p - g'p, \tag{1}$$

then the structural impedance can be developed like this:

$$Z(\omega) = \frac{(1+\mathrm{i}\eta)D_f k^6 + (1+\mathrm{i}\eta)g(D_f + D)k^4 - m\omega^2 k^2 - m\omega^2 g(1-\nu^2)}{\mathrm{i}\omega(k^2+g)}. \tag{2}$$

The TL can then be simply calculated with Z obtained for a certain acoustic angular frequency ω. In Equation (2), supposing that the geometric parameters are all pre-determined and k defined as the wavenumber, the other variables can be related to 5 basic mechanical parameters: E the faceplate Young's modulus, G_{xz} the core shear modulus in y-axis, G_{yz} the core shear modulus in x-axis, m the structural area density and η the structural damping factor. Among them, E and mostly contributes to D and D_f the structural and faceplate stiffness while G_{xz}, G_{yz} and η mainly contribute on g the core shear stiffness factor.

3 MESO-STRUCTURE AND ITS MECHANICAL PROPERTIES

The meso-structure is a general conception for the porous design of sandwich panel's core layer, among which the honeycomb structure is the most often seen, as presented in Figure 1.

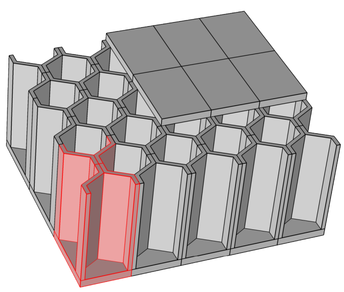

Figure 1. Sandwich panel with double vertical thickness honeycomb meso-structure

Limited to analytical models, only a small part of regular meso-structures can have simple expressions, including the double vertical thickness honeycomb structure, whose accurate analytical model is recently given by Malek and Gibson [5]. With this, we can evaluate the approximate homogenized structural parameters G_{xz}, G_{yz} and m with some given geometric values.

So, as G_{xz}, G_{yz} and m are all outputs of the meso-structural model, some correlations may exist among them while E and η are preset to be independent. And here is the correlation matrix obtained for this honeycomb structure:

$$\begin{array}{c} \\ eta \\ G_{xz} \\ G_{yz} \\ m \\ \eta \end{array} \begin{array}{c} \begin{array}{ccccc} E & G_{xz} & G_{yz} & m & \eta \end{array} \\ \begin{bmatrix} 1 & -0.00 & -0.00 & 0.01 & -0.00 \\ -0.00 & 1 & 0.68 & 0.51 & 0.02 \\ -0.00 & 0.68 & 1 & 0.61 & 0.01 \\ 0.01 & 0.51 & 0.61 & 1 & 0.01 \\ -0.00 & 0.02 & 0.01 & 0.01 & 1 \end{bmatrix} \end{array}, \quad (3)$$

which indicates that the three variables are very positively correlated, especially for the two shear modulus.

4 GLOBAL SENSITIVITY ANALYSIS WITH CORRELATED VARIABLES

In the domain of GSA, ANOVA (ANalysis Of VAriance) is one of the most mentioned system of SA algorithms. It's established on a high dimension expansion of the output's variance, supposing the model $Y = f(x_1, x_2, ..., x_n)$, it could be uniquely decomposed into this form:

$$V(Y) = \sum_i V_i(x_i) + \sum_i \sum_{j>i} V_{ij}(x_i, x_j) + \sum_i \sum_{j>i} \sum_{l>j} V_{ijl}(...) + ... + V_{123...n}(x_1, ..., x_n). \quad (4)$$

Therefore, the definition of the first order sensitivity index is $S_i = V_{X_i}(E_{X_{\sim i}}(Y|X_i))/V(Y)$, where $X_{\sim i}$ means all the inputs except X_i. The index S_i represents the ratio of variance of the output Y explained by the input X_i. Higher the value is, more important the uncertainty control is for this variable.

In this research, the sensitivity analysis methods applied belong to Fourier Amplitude Sensitivity Test (FAST) series, which have great advantage in calculation efficiency of first order sensitivity indices. Its classical version the FAST was realized by Saltelli and Bolado [6], but this one can not take variables's correlation properties into consideration. So two advanced algorithm with correlation are applied for comparison purpose, including the FASTC (proposed by Xu and Gertner [7]) and the FAST-orig (proposed by author).

5 TEST RESULTS

As shown in Figure 2, three SA algorithms are applied on the Mead's sound transmission model with correlated variables generated by the double vertical thickness honeycomb meso-structure for the frequency band from 100 to 10000Hz. Regarding the general form of these SI curves, some basic acoustic knowledge can be verified: the dominant role of m the mass per area at low frequency, and the importance of shear effect for mid-high frequency sound wave isolation.

Comparing the SA results obtained by FAST methods with (FAST-orig, FASTC) and without (FAST) correlation design, some interesting phenomena can be observed. With strong positive correlation among G_{xz}, G_{yz} and m, m becomes also significantly important at high frequency while G_{xz} and G_{yz} are no longer negligible at low frequency. Stroked by their increase of SI, η's importance is greatly compressed but no obvious change can be observed for SI(E). It's interesting to mention that though the mean value of G_{xz} and G_{yz} have huge difference because of the double vertical thickness, their SI curves have almost the same form with or without correlation.

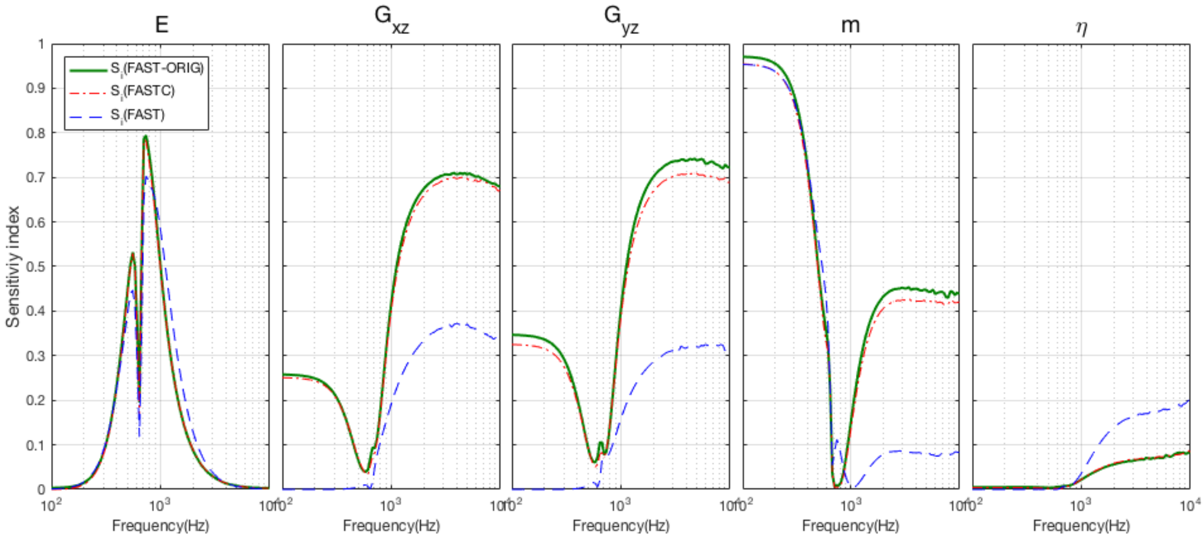

Figure 2. SI curves for the 5 mechanical parameters on the transmisson loss

6 CONCLUSION

In this research, some interesting features are observed when GSA methods are applied on an acoustic sandwich panel model with correlated mechanical parameters. As the correlation properties are generated by the sandwich core layer's meso-structures, it might be very interesting if various meso-structures can be tested. Also, for the sake of calculation efficiency, analytical models are chosen to be evaluated in GSA progress, some more convincible results could be obtained if finite element simulation or even experimental data can be directly taken into application.

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