Vibration reduction of composite plates with shunted piezo-patches: Analytical modeling and numerical validation

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ABSTRACT

Laminated composite plates are frequently used in various aerospace, automotive and space applications. In such applications, undesired vibrations can be reduced by using active and passive vibration control methods. Integrating piezo-patches into such plates is an alternative way to damp the structural vibrations. However, accurate models are required to predict the dynamics of such plates when the piezo is attached to the host structure. In this paper, analytical modeling of composite plates with two surface-bonded piezo-patches in parallel shunted to a resistive load is presented using Rayleigh-Ritz method. The analytical model accounts for mass and stiffness contribution of piezo-patches as well as two-way electromechanical coupling effect. Furthermore, electromechanical frequency response functions obtained by the developed analytical model are validated by finite-element analysis in ANSYS. Finally, the performance of the shunt damping method on the composite structure for a range of resistive loads is demonstrated by the reduction in the displacement frequency response functions.

1 INTRODUCTION

Thin composite plates are of interest in many engineering applications, for their superior mechanical characteristics such as high stiffness-to-weight ratio and low density. However, in harsh environments, they are exposed to severe vibrations which result in reduced structural life and eventually mechanical failure [1]. Piezoelectric patches integrated into such flexible structures can be utilized to passively damp the vibrations through shunting the piezo-patch electrodes with an electrical circuit [2,3]. Few analytical models have been proposed in the literature for vibration analysis of thin plates with surface-bonded piezo-patches [4,5]. In the reported works, only piezoelectric coupling effect is included in the analytical modeling, whereas mass and stiffness contribution of piezo-patches are ignored due to the low volumetric ratio of piezo-patches with respect to the isotropic plate (made of Aluminium). This assumption, however, does not remain valid for lightweight composite plates. Therefore, in this study, the effect of piezoelectric shunt damping on the composite plates is investigated through analytical modeling that accounts for mass and stiffness properties of the piezoelectric material and two-way electromechanical coupling. Moreover, the analytical model results and vibration reduction of the host structure using the piezo patches is validated by finite-element simulations in ANSYS.

2 ANALYTICAL MODELING

In this section, equations of motions for a composite plate with two surface-bonded piezo-patches are presented. Figure 1 shows the schematics of the electromechanical system where a pair of piezo-patches are connected in parallel to a resistive load.

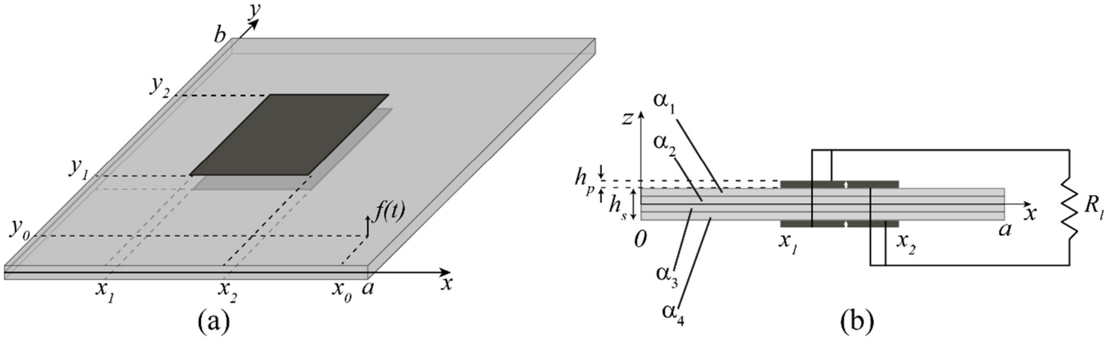

Figure 1. (a) schematics of a composite plate with two surface-bonded piezo-patches, (b) cross-section view of the laminated composite (with four ply angles α_1, α_2, α_3, α_4) where piezo-patches are shunted in parallel to a resistive load.

Applying Hamilton's principle and using Rayleigh-Ritz method for modal analysis procedure [6], we obtain the following electromechanical equations for the two structurally integrated piezo-patches connected in parallel to a resistive load in the modal coordinates:

$$\frac{\mathrm{d}^2\eta_{mn}(t)}{\mathrm{d}t^2}+2\zeta_{mn}\omega_{mn}\frac{\mathrm{d}\eta_{mn}(t)}{\mathrm{d}t}+\omega_{mn}^2\eta_{mn}(t)-v(t)\sum_{k=1}^{2}(\tilde{\theta}_{mn})_k=f_{mn}(t) \tag{1}$$

$$\sum_{k=1}^{2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}(\tilde{\theta}_{mn})_k\frac{\mathrm{d}\eta_{mn}(t)}{\mathrm{d}t}+\frac{\mathrm{d}v(t)}{\mathrm{d}t}\sum_{k=1}^{2}(C_\mathrm{p})_k+\frac{v(t)}{R_l}=0 \tag{2}$$

where $\eta_{mn}(t)$ and $v(t)$ are the mechanical modal coordinate and voltage over across the resistive load. Here, $(C_{\mathrm{p}})_k$ is the equivalent capacitance for the kth patch and given by $(C_{\mathrm{p}})_k = (\bar{\varepsilon}_{33}^{\mathrm{S}})_k (A_p)_k / (h_{\mathrm{p}})_k$ (where $\bar{\varepsilon}_{33}^{\mathrm{S}}$, A_p, and h_p are the dielectric permittivity, the piezo-patch area and thickness in z-direction, respectively). The Modal forcing input and the electromechanical coupling term can be expressed as:

$$f_{mn}(t) = f(t) U_{mn} W_{mn}(x_0, y_0) \tag{3}$$

$$(\tilde{\theta}_{mn})_k = \left(\frac{h_p + h_s}{2}\right)(\bar{e}_{31})_k \int_{y_1}^{y_2}\int_{x_1}^{x_2} U_{mn}\left[\frac{\partial^2 W_{mn}(x,y)}{\partial x^2} + \frac{\partial^2 W_{mn}(x,y)}{\partial y^2}\right]\mathrm{d}x\mathrm{d}y \tag{4}$$

here, $\bar{e}_{31}$ is the effective piezoelectric constant, and $U_{mn}W_{mn}(x,y)$ are the assumed modes where $m = 1,2,\ldots,M$, $n = 1,2,\ldots,N$ (*M, N* indicate the number of modes).

It should be noted that mass and stiffness effect of piezoelectric patches are included in the modal analysis procedure using the following indicator function $P(x,y)$, which determines the area on the plate that is covered by the piezo-patch as

$$P(x,y) = \left[H(x-x_1) - H(x-x_2)\right] \times \left[H(y-y_1) - H(y-y_2)\right] \tag{5}$$

where *H* denotes the Heaviside unit-step function.

For the electromechanical frequency response functions (FRFs), one can substitute the harmonic forms of $f(t) = F_0 e^{j\omega t}$, $\eta_{mn}(t) = H_{mn} e^{j\omega t}$, and $v(t) = V e^{j\omega t}$ at steady-state into Equations (1) and (2), and obtain the displacement output to force input FRF as follows:

$$\beta(x,y,\omega) = \frac{w(x,y,t)}{F_0 e^{j\omega t}}, \quad w(x,y,t) = \sum_{m=1}^{M}\sum_{n=1}^{N} U_{mn} W_{mn}(x,y)\eta_{mn}(t) \tag{6}$$

3 RESULTS AND DISCUSSIONS

Figure 2 shows the electromechanical frequency responses (voltage and displacement outputs per unit force input) of the coupled system for validation of the proposed analytical model.

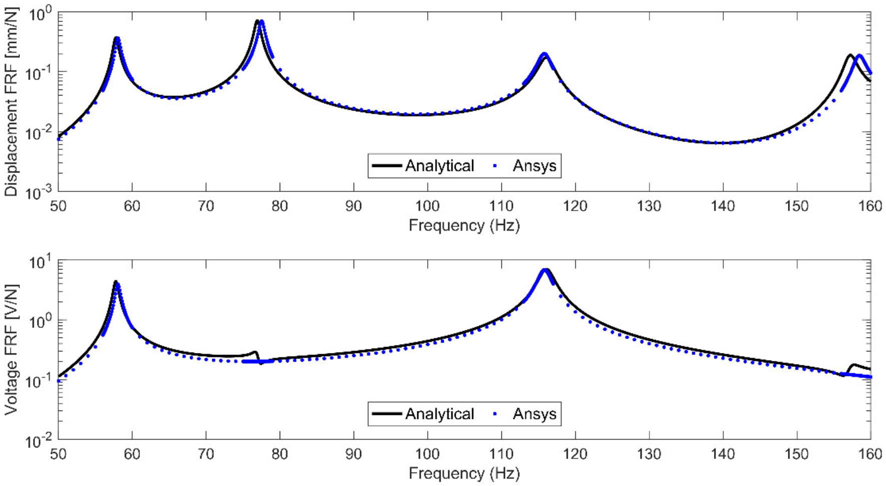

Figure 2. Comparison of Analytical voltage and displacement FRF with the corresponding ANSYS simulations.

As it can be observed from the graph, the analytical solution based on Rayleigh-Ritz method agrees well with the ANSYS simulation results. The electrical resistive load value is taken as 1 kΩ in the presented FRFs. The vibration response shown in Figure 2 is measured at the center of the top-left quarter of the plate. The voltage FRF shows a cancellation for the second and fourth modes since the piezo-patches are located in the center of the plate. These charge cancellations can be explained by the in-phase and out-phase strain distribution that occurs in the area of the plate covered by the patches. This result show that, one should carefully choose the location of the patches for effectively reducing the vibrations at the target frequency such that no electrical cancellation occurs.

4 CONCLUSION

An analytical modeling of a laminated composite plate with surface bonded piezoelectric patches, in bimorph configuration, using Rayleigh-Ritz method was presented. Analytical solutions for electromechanical FRFs were validated against ANSYS simulation results. The analytical model accounts for mass and stiffness effect of piezo-patches on the composite plate along with two-way electromechanical coupling effect. It was shown that using the analytical model, one can correctly predict the vibration response of the plate in presence of shunted piezo-patches. Furthermore, an optimum resistance value can be obtained for maximum reduction of vibration level at a target excitation frequency.

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