NONFACTORIZABLE VISCOELASTIC BEHAVIOR: MODELING AND IDENTIFICATION

A. Tayeb[1,2] ,A. Zine[3] ,M. Arfaoui[1], M. Ichchou[2*], A. Hamdi[1] and J. Benabdallah[1]

[1] LR-11-ES19 Laboratoire de Mcanique Appliquée et Ingénierie
Université de Tunis El Manar, École Nationale dIngnieurs de Tunis
Tunis 1002, Tunisie
Email: teyebadel@yahoo.fr, makremarfaoui@yahoo.fr, ahamdi.tn@gmail.com and jalel.benabdallah@enit.rnu.tn

[2]Laboratoire de tribologie et dynamique des systèmes
'Ecole Centrale de Lyon, Ecully 69130, France
Email: mohamed.ichchou@ec-lyon.fr

[3]Université de Lyon, Institut Camille Jordan, CNRSUMR5208, Département de Mathématiques et Informatique
'Ecole Centrale de Lyon, Ecully 69130, France
Email: abdel-malek.zine@ec-lyon.fr

ABSTRACT

A three dimensional viscoelastic model at finite strain representing nonfactorizable behaviour of rubber like materials is proposed. The model is based upon the internal state variables approach within the framework of rational thermodynamics such that the second principle of thermodynamics is satisfied. Motivated by experimental and rheological results, the nonfactorizable aspect of the behavior was introduced via strain dependent relaxation times which leads to a reduced time with a strain shift function. The identification of the models parameters and its capacity to predict the nonfactorizable behaviour of rubber like materials with the multi-integral viscoelastic model of Pipkin is addressed.

1 INTRODUCTION

It is well known that rubber-like materials exhibit nonlinear viscoelastic behavior over a wide range of strain and strain rates confronted in several engineering applications such as civil engineering, automotive and aerospace industries. Further, the time dependent properties of these materials, such as shear relaxation modulus and creep compliance, are, in general, functions of the history of the strain or the stress [1]. Therefore, in a wide range of strain a linear viscoelasticity theory is no longer applicable for such material. Hence, new constitutive equations are required to fully depict the behavior of rubber-like. In this work we shall develop a nonlinear model at finite strain for nonfactorizable viscoelastic materials within the framework of rational thermodynamics and the approach of internal state variables, see [2], [3] and [4] taking into account the dependence of the time dependent functions upon the state of the strain. The identification of several functions in the model to the multi-integral model of Pipkin [5] is performed with Matlab software. This paper is organized as follows: in section 2 the mechanical framework and the model are recalled. In section 3 a brief review of the model by [5] is presented and the results of the identification are highlighted.

2 MECHANICAL FRAMEWORK AND CONSTITUTIVE EQUATIONS

Consider a viscoelastic material with reference placement Ω_0 in the reference configuration C_0. It occupies at the time t the placement Ω in the deformed configuration C_t. Let φ denote a macroscopic motion between the two configurations, which maps any point X in the reference configuration C_0 to the point x in the deformed configuration. Let $F\left(X,t\right) = \partial x/\partial X$ be the deformation gradient tensor. Likewise, let $J = \det\left(F\right)$ be the jacobian of the deformation gradient tensor. From the deformation gradient $F(X,t)$, the right and left Cauchy-Green strain tensors $C = F^tF$ and $B = FF^t$ are obtained. The formulation of the constitutive equations in the nonlinear range of behavior is based upon the decomposition of the deformation gradient tensor $F(X,t)$ into volumetric and isochoric parts such that:

$$\bar{F} = J^{-1/3}F \qquad \text{where } \det\left(\bar{F}\right) = 1 \tag{1}$$

in which $\bar{F}$ is the isochoric part of the deformation gradient tensor, the right and left Cauchy-Green strain tensors associated with it reads:

$$\bar{C} = \bar{F}^t\bar{F} = J^{-2/3}C, \;\; \bar{B} = \bar{F}\bar{F}^t = J^{-2/3}B \tag{2}$$

The free energy density according to [2] is expressed as follows:

$$\Psi\left(\bar{C},Q\right) = U^0\left(J\right) + \bar{\Psi}^0\left(\bar{C}\right) - \frac{1}{2}Q:\bar{C} + \Psi_I\left(Q\right) \tag{3}$$

in which Q is a second order tensor internal variable akin to the second Piola-Kirchhoff stress tensor, its evolution law is expressed as follow:

$$\frac{\partial Q}{\partial \xi} + \frac{1}{\tau}Q = \frac{\gamma}{\tau}DEV\left[2\frac{\partial\Psi^0\left(\bar{C}\right)}{\partial\bar{C}}\right] \qquad \text{with } \xi\left(t\right) = \int_0^t \frac{dt'}{a\left(\bar{C}\right)} \tag{4}$$

in which $DEV\left(\bullet\right) = \left(\bullet\right) - \frac{1}{3}\left[C:\left(\bullet\right)\right]C^{-1}$ denotes the deviator operator in the reference configuration. γ and τ are the viscoelastic parameter and the relaxation time of the Prony series respectively, in relation 4 ξ denotes the reduced time which is an increasing function of real time t and $a(\bar{C})$ is a positive function of the left Cauchy-Green strain tensor called a strain-shift function. Application of the Clausius-Duhem inequality and the resolution of the evolution equation 4 along with the form of the free energy density of equation 3 lead to the expression of the second Piola-Kirchhoff stress tensor.

$$S = J^{-2/3} \int_0^{\xi} G\left(\xi - \xi'\right) \frac{\partial}{\partial \xi'} DEV \left(2 \frac{\partial \Psi^0 \left(\bar{C}\right)}{\partial \bar{C}} \right) d\xi' + JpC^{-1} \tag{5}$$

3 IDENTIFICATION OF THE PIPKIN MODEL

3.1 Pipkin isotropic model

Pipkin [5] proposed a third order development of the tensorial response function Q for an isotropic incompressible material. The principle of material indifference requires that the Cauchy stress tensor takes the following form:

$$\sigma = RQR^t + pI \tag{6}$$

R is the rotation tensor obtained from the polar decomposition of the transformation gradient tensor F and p is the indeterminate parameter due to incompressibility. The third functional development of Q reads

$$\begin{aligned} Q\left(t\right) = &\int_0^t r_1\left(t-t'\right)\dot{E}\left(t'\right)\, dt' + \int_0^t \int_0^t r_2(t-t', t-t'')\dot{E}\left(t'\right)\dot{E}\left(t''\right) dt'\, dt'' + \\ &\int_0^t \int_0^t \int_0^t r_3\left(t-t', t-t'', t-t'''\right)\, tr\left[\dot{E}\left(t'\right)\dot{E}\left(t''\right)\right]\dot{E}\left(t'''\right)\, dt'dt''dt''' + \\ &\int_0^t \int_0^t \int_0^t r_4\left(t-t', t-t'', t-t'''\right)\, \dot{E}\left(t'\right)\dot{E}\left(t''\right)\dot{E}\left(t'''\right)\, dt'dt''dt''' \end{aligned} \tag{7}$$

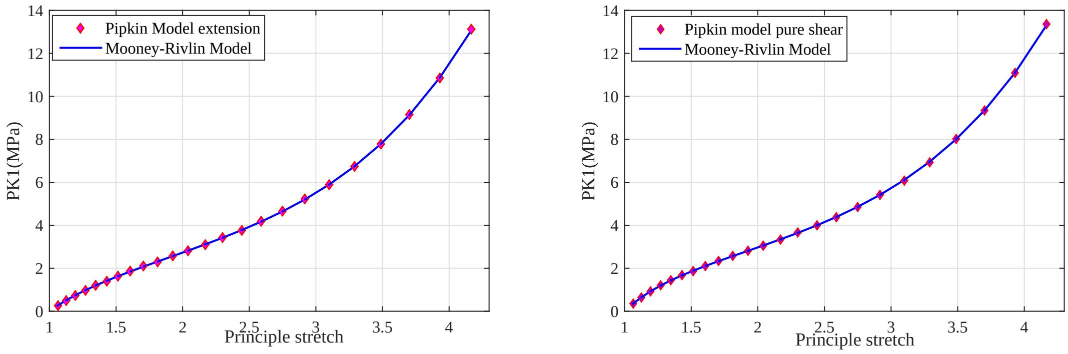
$r_i\, i = 1...4$ are the relaxation kernels expressed by a decaying exponential functions and $\dot{E}\left(t\right)$ is the time derivative of the Green-Lagrange deformation tensor $E = 1/2\left(C - I\right)$.

3.2 Identification of the model's functions

The free energy density Ψ^0, the viscoelastic kernel $G(\xi)$ and the reduced time $\xi(t)$ of relation 5 are identified separately. To this end data in pure shear and simple extension were generated following relations 6 and 7. Equilibrium tests of simple extension and pure shear are used in the identification of Ψ^0, relaxation tests with small level of strain in pure shear are used in the identification of $G(\xi)$ and monotonic tests of simple extension are used in the identification of $\xi(t)$ and then the whole identification procedure is validated by predicting the response of the model to a monotonic test of pure shear. Each identification procedure turns out to a least square minimization problem. The results of this identification are plotted in figure 1 in terms of the hyperelastic response and in figure 2 in terms of the reduced time function and the predicted response of the model in pure shear for two different strain rates: $\dot{\varepsilon} = 100\%$ and $\dot{\varepsilon} = 200\%$.

4 CONCLUDING REMARKS

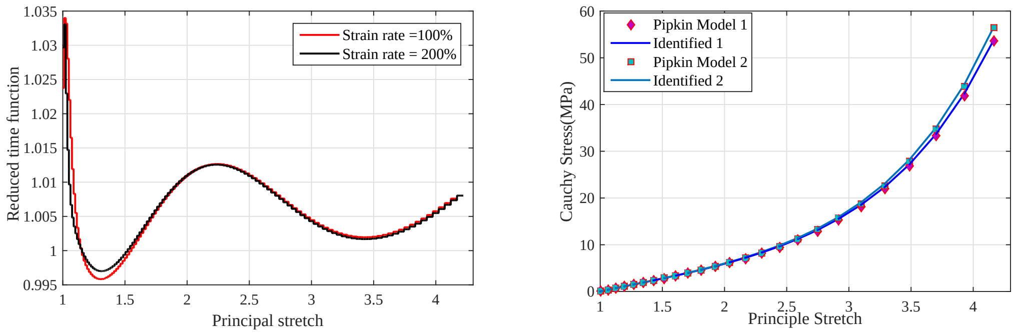
A nonlinear viscoelastic model at finite strain to describe nonfactorizable behavior of rubber like materials has been proposed. The model is formulated using the decomposition of the deformation gradient tensor which makes it applicable to both compressible and incompressible materials. The identification of the model's functions to the multi-integral isotropic model of Pipkin [5] is highlighted and a significant potential of the model to track the response of this model is obtained.



(a) Piola-Kirchhoff stress versus principle stretch for simple extension

(b) Piola-Kirchhoff stress versus principle stretch for pure shear

Figure 1. Equilibrium response for the Pipkin model and the Mooney-Rivlin model [6]



(a) Reduced time function $a(\bar{C})$ versus principle stretch for two strain rates

(b) Cauchy stress in simple shear versus principle stretch for two strain rates

Figure 2. Identification results: Reduced time function and Cauchy stress

REFERENCES

[1] R. A. Schapery. An engineering theory of nonlinear viscoelasticity with applications. *International Journal of Solids and Structures*, 2(3):407–425, July 1966.

[2] R. A. Simo. On a fully three-dimensional finite-strain viscoelastic damage model: Formulation and computational aspects. *Computer Methods in Applied Mechanics and Engineering*, 60:153–173, 1987.

[3] G. Holzapfel and R. A. Simo. A new viscoelastic constitutive model for continuous media at finite thermomechanical changes. *International Journal of Solids and Structures*, 33:3019–3034, 1996.

[4] S. Reese and S. Govindjee. A theory of viscoelasticity and numerical applications. *International Journal of Solids and Structures*, 35:3455–3482, 1998.

[5] A. C. Pipkin. Small finite deformations of viscoelastic solids. *Reviews of Modern Physics*, 36(4):1034–1041, 1964.

[6] R.S. Rivlin. Large elastic deformations of isotropic materials. iv. further developments of the general theory. *Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 241(835):379–397, 1948.