FIRST LITERATURE REVIEW FOR THE ANALYSIS OF QUASI-PERIODICITY VARIABILITY EFFECTS AND MODELING STRATEGIES

S. Timorian[1], S. De Rosa[1], F. Franco[1], M. Ouisse[2] and N. Bouhaddi[2]

[1]PASTA-Lab / Laboratory for promoting experiences in Aeronautical Structures and Acoustics,
Department of Industrial Engineering, University of Napoli "Federico II "
Via Claudio 21, 80125 Napoli, Italy
safiullah.timorian@gmail.com

[2]Univ. Bourgogne Franche-Comte, FEMTO-ST Institute, CNRS/UFC/ENSMM/UTBM,
Department of Applied Mechanics, 25000
BESANCON, FRANCE

ABSTRACT

Quasi periodicity and variability effects on structures are spread in all the engineering branches. The inherent research works are carried out with both numerical and experimental activities. A more specific goal is the analysis of the influence of such effects on the vibroacoustic response. The modern tools for investigating the most significant impacts of imperfections and irregularities on the vibrational and acoustical response of a given structure are codes based on combination of wave and finite element, spectral finite elements, transfer matrix methods and the adoption of stochastic variables. In view of the foreseen work inside the VIPER project, this paper refers to the analysis of the most recent literature and possible initial strategy.

1 INTRODUCTION

"*Quasi-periodicity is the property of a system that displays irregular periodicity. Periodic behaviour is defined as recurring at regular space and time intervals. Quasi-periodic behaviour is a pattern of recurrence with a component of unpredictability that does not lend itself to precise measurements*": this simple definition is taken from Wikipedia.

It precisely points to the core of the problem. In fact, an increasing literature is appearing on methods for the analysis of a given system by replicating only its elemental cell in space directions and time scale, thus simulating conditions of perfect periodicity. How to simulate systems, if perfect periodic conditions are violated, is still to be analysed together with their influence.

This paper belongs to the VIPER project (http://viper.ec-lyon.fr.) which is fully centred on the vibroacoustic of periodic media. For the periodic structures, the definitions of the effect of quasi-periodicity is to be investigated in order to understand the physics, how this can be modelled and what are the effects of the final design. In fact, it will be important to analyse if and how the presence of imperfections or irregularities, on quasi-periodic bases, can have a significant impact on the vibro-acoustic responses of given components.

It is expected and already shown that the effects on micro-scale can influence the performance on macro-scale: the engineering design can receive important information if more light is shed in this link. Several effects are known in literature (the Anderson localization is one of the most famous) but some insights are now necessary in order

- *to **improve the simulations** of these kind of problems;*
- *to **move to prototypes** which can demonstrate on experimental basis the achievement of increased vibroacoustic performances (structural damping and/or acoustic transmission loss).*

2 LITERATURE REVIEW

Quasi periodic structures are scattered from those identical cells which has a periodic form in the assembly of a piece of element in the structures. The analysis could be done by the presence of imperfection or irregularity which have a significant impact on the vibrational and acoustic behaviour.

The difference between imperfection or irregularity in a quasi-periodic sequence has to be defined: the importance of this step should not be underestimated.

In [1], an enriched finite element method is presented to solve various wave propagations. However, the standard finite element method is not very effective for utilising the solution of the wave propagation problems [2]. The errors introduced in this method have been identified and analysed and they are due to the fact the wave propagation analysis is based on piecewise polynomial approximation: the accuracy of the numerical solution becomes rapidly worse with increasing wave number.

Ref. [3] and [4] are a good example of how the use of periodicity and the Wave and Finite Element can be real enhancements of the predictive quality.

An extension of WFEM is presented in [5]: the actual status of the wave and finite element method has been identified as a best approach for the vibroacoustic analysis over periodic structures. The numerical method has been carried out via Bloch's theorem and imposing periodicity conditions to a single cell which represents a repetitive part of the whole structure. The results show a promising agreement between Wave and Finite Element Method (WFEM) and Classical Finite Element Method (CFEM). Experimental testing and validating comparisons on stiffened cylinders are ongoing at Pasta-Lab.

It has to be remembered that the prototype under research and investigation is a structural component that has characteristics which periodically repeat in one or more directions. In general, a generic structure obtained as an assemble of identical elements, called *cells*, which can be considered as periodic structures. The modelling of a short section of the waveguide is expressed by supposing time harmonic motion, the equation of motion is implied by discrete coordinates, relating nodal degrees of freedom q and force f of the undamped section:

$$(K - \omega^2 M)q = f \tag{1}$$

where K and M are the stiffness and mass finite element matrices, [4-8]. The analysis of periodic systems is thus well undertaken through WFEM.

The analysis of transmission and diffusion at joints between waveguides as well as the damped periodical waveguides were investigated in several research works, [7-8].

In [9-11], the first models to take into account the irregularities are presented: they are very recent and represent useful investigations to guide the next required steps.

At the moment, the approach adopted in [3] seems the most promising for reproducing the forced response of periodic structures in presence of quasi-periodicity and variability effects. The flexibility of getting the K and M matrices from standard finite element codes and the definition of an external post-processing code appears as the most viable procedure even in view of the expected variations to be included for simulating the quasi-periodicity.

3 STRATEGY

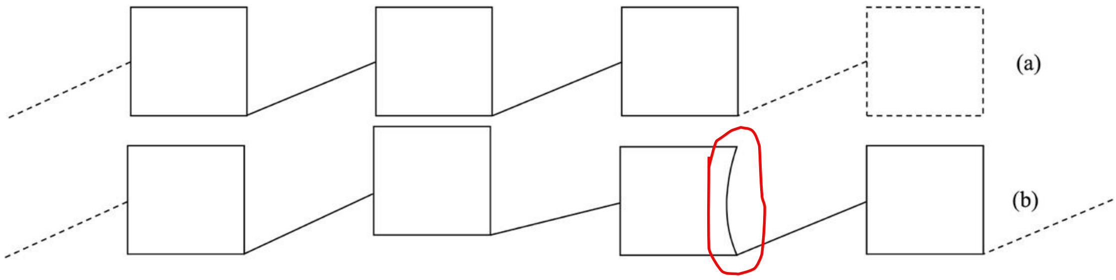
The envisaged steps for facing a such complicated problem are the following:

A. definition of the quasi-periodicity;
B. analysis of the nature and sizes of the causes altering the perfect periodicity;
C. definition of the required
    1. *mathematical,*
    2. *numerical and*
    3. *experimental tools.*

They are all challenging. In Fig. 1 a sketch of the possible problems/configurations is reported: the perfect periodic system (a) can be altered to get a (b) quasi periodic system in terms of shapes, junctions, sizes, materials, manufacturing issues, etc.; what does *quasi* mean?

These effects could be evaluated adding each of them on a predictive environment: this will lead to development of a new class of codes expected to be based on a combination of wave and finite element, spectral finite element, transfer matrix methods and the adoption of stochastic variables, if needed. It has to be considered also the possibility to use non-deterministic (*possibilistic*) algebras as those associated to the fuzzy-logic or the interval algebra.

In literature, it is already shown that a sandwich panel, optimized versus the vibroacoustic performance, with added random properties of the core can exhibit stop-band characteristics in some frequency ranges. Therefore, the analysis of these quasi-periodicity effects is much closer to the engineering application than many others trying to simulate real structures with 3D models with huge computational costs.



(a). Periodic System, (b). Quasi-Periodic System
Figure 1: Sketches of the periodic and *quasi*-periodic distribution of repeated cells.

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