SIMULATION OF FINITE-SIZED DYNAMIC SYSTEMS USING WAVE TRANSMISSION METHODS

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ABSTRACT

For predicting machinery noise in operating conditions over the audio frequency range, statistical methods are required. The most popular is the Statistical Energy Analysis (SEA). Originally based on modal response of coupled oscillators, this method has become more a wave approach in diffuse field condition than a modal kind of analysis. This is due to basic assumption of weak coupling between subsystems that is still the drawback of SEA method. SEA parameters of a built-up system cannot directly be obtained from deterministic modal formulation and have to be inferred from measurement or numerical FEM modeling by inverse techniques. In SEA, all internal calculated parameters are wave-based parameters thanks to some a priori assumption on system behavior. These parameters are extracted from simple infinite or semi-infinite propagating wave models. As the system under analysis is bounded and generally weakly damped, it is necessary to tune infinite wave parameters to fit to finite systems. Series of examples are presented to illustrate the observed differences between finite and infinite systems and how to correct the infinite calculation to get realistic parameters for finite-sized systems.

1 INTRODUCTION

Acoustic transmission loss predictions are performed at low cost with the simplified Transfer Matrix Method (TMM). In TMM, layers are assumed isotropic and of infinite extent with finite small thickness in transverse direction. When gluing several layers on top of each other, the resulting system may be inserted within two semi-infinite acoustic volumes as shown in Figure 1. The sound transmission from a double-walled configuration may then be predicted by TMM representation by inserting an acoustic layer between two solid elastic layers. Sending an acoustic emitter wave impinging solid elastic layer 1, this wave will be partly reflected and partly transmitted through the different layers and will exit in the receiver fluid after some attenuation.

The infinite extent of layers may cause artefact in the noise transmission prediction as shown in next paragraph. Finite-sized corrections are then introduced to make the prediction more regular with better test fitted results.

2 MASS LAW SOUND TRANSMISSION USING INFINITE LAYER

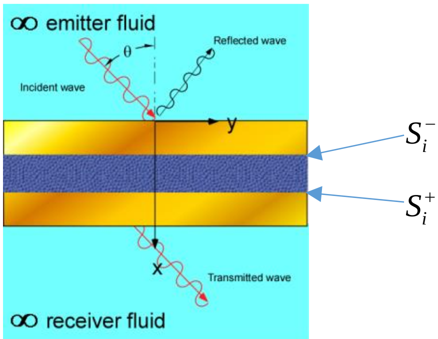

Figure 1. An acoustic sound package modelled as TMM.

For predicting this attenuation, the laws under which stress and displacement are transferred from one layer to another need to be given. These laws depend on layer type.

The simplest kind of layer is the pure inertial mass layer which oscillates without any residual stiffness. The mass layer is assumed uncompressible and is defined by ρ_s the mass density of the layer per m². The incident wave amplitude has constrained harmonic motion along *y* axis with acoustic potential defined as:

$$\varphi(x, y, \omega) = e^{-ik\cos\theta x} e^{-ik\sin\theta y} \tag{1}$$

from which pressure and velocity are derived:

$$p = i\omega\rho_0\varphi \qquad v = -\frac{\partial\varphi}{\partial x} \tag{2}$$

On emitter side, the wave is reflected while on receiver side the wave is simply transmitted. Emitter and receiver potentials are therefore given by:

$$\varphi_E = \left(e^{-ik\cos\theta x} + Re^{ik\cos\theta x}\right)e^{-ik\sin\theta y} \;; \quad \varphi_R = Te^{-ik\cos\theta x} e^{-ik\sin\theta y} \tag{3}$$

with *R* and *T* resp. reflection and transmission wave amplitudes.
At each layer boundaries indexed by *i*, pressure and velocity are submitted to change, defining two state vectors both sides of the layer S_i^- and S_i^+ such as

$$S=\begin{bmatrix} v \\ -p \end{bmatrix}=\begin{bmatrix} v \\ \sigma \end{bmatrix} \tag{4}$$

where σ is the normal stress, opposite sign of the pressure.
Continuity of *S* at interface between two layers is also assumed: $S_i^+ = S_{i+1}^-$. Downward velocity, v^- and upward one, v^+ are then equal. The stress variation both sides of this layer is equal to the inertial force $\rho_s \frac{\partial v}{\partial x}$. It leads to the two following equations:

$$\left.\begin{aligned} & v^+ - v^- = 0 \\ & \sigma^+ - \sigma^- - \rho_s \frac{\partial v}{\partial t} = 0 \end{aligned}\right\} \text{and in matrix form } \begin{bmatrix} v^+ \\ \sigma^+ \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \rho_s \frac{\partial}{\partial t} & 1 \end{bmatrix} \begin{bmatrix} v^- \\ \sigma^- \end{bmatrix} = Z \begin{bmatrix} v^- \\ \sigma^- \end{bmatrix} \tag{5}$$

The Z-matrix is actually transferring the downward information contained in the state vector S^- to upward one S^+. Reversely $S^- = Z^- S^+$. In the case *N* layers are connected to each other's, if S_E is the source vector in the emitter fluid, with same state vector variables for each layer, the state vector in the receiver fluid is obtained as:

$$S_R = Z^{-1}{}_N Z^{-1}{}_{N-1} .. Z^{-1}{}_1 S_E \tag{6}$$

To fully solve this problem, S_E and S_R need to be expressed as function of *R* and *T*.
From (3), (4), (5) and (6), S_E and S_R are obtained:

$$S_E = ik \begin{bmatrix} \cos\theta & -\cos\theta \\ -\rho_f c_f & -\rho_f c_f \end{bmatrix} \begin{bmatrix} 1 \\ R \end{bmatrix} \qquad S_R = ik \begin{bmatrix} \cos\theta \\ -\rho_f c_f \end{bmatrix} T \tag{7}$$

Using (5) and (6) in frequency domain where $\frac{\partial}{\partial t} \to i\omega$, it comes:

$$\begin{bmatrix} \cos\theta \\ -\rho_f c_f \end{bmatrix} T = \begin{bmatrix} 1 & 0 \\ -i\omega\rho_s & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\cos\theta \\ -\rho_f c_f & -\rho_f c_f \end{bmatrix} \begin{bmatrix} 1 \\ R \end{bmatrix} \tag{8}$$

We end up with two equations and two unknown's *T* and *R* of which solution is:

$$\left.\begin{aligned} & T = 1 - R \\ & T - \frac{i\omega\rho_s \cos\theta}{\rho_f c_f} - 1 + R\left(\frac{i\omega\rho_s \cos\theta}{\rho_f c_f} - 1\right) = 0 \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} & T = \frac{2}{2 - \frac{i\omega\rho_s \cos\theta}{\rho_f c_f}} \\ & R = 1 - T \end{aligned}\right. \tag{9}$$

T expression is called the acoustic mass law predicting the sound transmission between two acoustic volumes containing same fluid and separated by a pure thin inertial layer.

It is remarkable to note that *T* is equal to 1 at grazing incidence $\theta = \frac{\pi}{2}$ independently from mass density value, which is not the result we intuitively expect. If the incident wave is parallel to the mass layer, there is a non-zero pressure on the related surface of the mass layer but its normal velocity will be zero and then the layer would not radiate any pressure in the receiver fluid. The radiated pressure should then be null. There is no contradiction if we come back to the definition of

T: *T* refers to the amplitude of the receiver wave which is parallel to the mass layer. The radiated intensity by the mass layer is given by the product of pressure time velocity which is expressed by:

$$I_{rad} = p_R v_R^* \tag{10}$$

From (7) it comes:

$$I_{rad} = k^2 \rho_f c_f |T|^2 \cos\theta \tag{11}$$

The normal radiated power is then effectively null when $\theta = \dfrac{\pi}{2}$ as $T = 1$.

The transmission loss coefficient is commonly expressed as the ratio of radiated power over incident power normal to the panel and using (2) for incident pressure and velocity, it gives:

$$\tau = \frac{I_{rad}}{I_{inc}} = \frac{k^2 \rho_f c_f |T|^2 \cos\theta}{\omega \rho_f k \cos\theta} = |T|^2 \tag{12}$$

The definition of transmission coefficient is thus ill-suited for describing the effective noise transmission under grazing incidences when panel size is infinite. It leads to overestimate wave intensity originated from incidences near 90°. Due to constrained infinite motion in *y*-direction, the stress on receiver side has always to compensate the emitter-side stress under grazing incidence, leading to equal stress both sides at 90° incidence as no other force may exist in the layer to cancel out the emitter stress. This observation is also true for other kinds of layer (porous fluids or elastic plates) but mass layer is particularly affected by infinite assumption. This is known for a long time and empirical correction is used when dealing with random-incidence acoustic field [1]. The random-incidence transmission loss is calculated by averaging (12) over incidence and in the case of mass-law, the averaging is limited to angles not exceeding 78.5°. This empirical value provides an average transmission coefficient that fits better with experimental measurements.
To get more regular (i.e. more physical transmission loss prediction), we need to take into account the finite size of actual panels in deriving sound transmission loss expression.

3 FINITE-SIZED SOUND TRANSMISSION LOSS

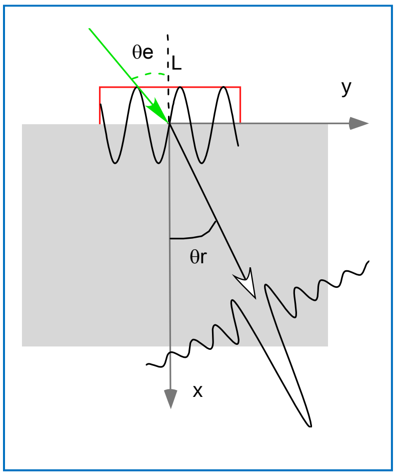

Figure 2. Convolution through aperture.

The junction interface is now considered with finite size L. As shown in Figure 2, a monochromatic emitter wave having wavenumber k_e and incidence θ_e, constrains the junction wavenumber in the in-plane direction y to follow the Descartes's law:

$$k_0 = k_e \sin\theta_e \tag{13}$$

In k_y-wavenumber space of the surface motion of the junction, the projected field of the incident wave is described by the distribution $\delta(k_y - k_0)$

The Spatial Fourier's Transform (SFT) of the velocity field *v(y)* on junction is noted by $\tilde{v}(k_y)$ and from Parseval's theorem, the radiated power of the junction may be then expressed as:

$$P_{rad}^F(\omega,\theta_r) = \int p(y)v^*(y)dy = \frac{1}{2\pi}\int Z_{rad}\tilde{v}(k_y)\tilde{v}^*(k_y)dk_y \tag{14}$$

The junction aperture is introduced as a weighting window defined by the Heaviside function such as: $-L/2 \le x \le L/2$, $w=1$ and $w=0$ elsewhere.

The SFT of w is given by $\tilde{w}(k_y) = L\sin\left[\frac{L}{2}k_y\right] / \frac{L}{2}k_y = L\,\mathbf{sinc}(k_y)$.

Assuming Z_{rad} weakly dependent of k_y, the monochromatic wave radiated power gives:

$$P_{rad}^F(\omega,\theta_r) = Z_{rad}\left|V_\infty\right|^2\int\delta(k_y - k_0)w^2(k_y)dk_y = P_{rad}^\infty\tilde{w}^2(k_y - k_0) \tag{15}$$

Intensity is thus expressed as:

$$I_{rad}^F(\omega,\theta_r) = I_{rad}^\infty\mathbf{sinc}^2(k_y - k_0) \tag{16}$$

The finite-sized transmission loss at a given incidence θ_e in the emitter is obtained from the infinite transmission loss under incidence α by the relationship:

$$\tau_{Finite}(\theta_e) = \frac{I_{n\_rad}^{finite}}{\rho_e c_e\cos\theta_e} = \frac{1}{\rho_e c_e\cos\theta_e}\frac{\int_0^{k_0} Z_f^\infty v^2\cos(\alpha)\left|\mathbf{sinc}(k_0 - k_y)\right|^2 dk_y}{\int_0^{k_0}\left|\mathbf{sinc}(k_0 - k_y)\right|^2 dk_y} \tag{17}$$

Pushing the reference incident intensity under the integrand, gives the relationship between the infinite and finite transmission loss.

$$\tau_{Finite}(\theta_e) = \frac{\int_0^{k_0}\frac{Z_f^\infty v^2\cos(\alpha)}{\rho_e c_e\cos\theta_e}\left|\mathbf{sinc}(k_0 - k_y)\right|^2 dk_y}{\int_0^{k_0}\left|\mathbf{sinc}(k_0 - k_y)\right|^2 dk_y} \approx \frac{\int_0^{\pi/2}\tau_\alpha^\infty\cos^2\alpha\left|\mathbf{sinc}(k_e\sin\theta_e - k_e\sin\alpha)\right|^2 d\alpha}{\int_0^{\pi/2}\cos(\alpha)\left|\mathbf{sinc}(k_0 - k_y)\right|^2 d\alpha} \tag{18}$$

The finite transmission loss is therefore obtained as a cosinus/sinc²-weighted average of the infinite transmission loss. This correction is called ***Convolution*** correction.

A more resource-demanding correction has been proposed in [3] and considers the finite window aperture (without the layer) as an additional transmission loss with specific radiation efficiency.

A single specular wave impinging the aperture with $\theta_0 - incidence$ is carrying an incident intensity:

$$I_0 = \frac{p_0^2}{\rho_0 c_0}\cos\theta_0 \tag{19}$$

When the layer is infinite, the radiated power is expressed as:

$$I_{rad}^{\infty} = \frac{p_r^2}{\rho_r c_r}\cos\theta_r \tag{20}$$

The sound transmission loss coefficient in finite condition, τ_F, is written as function of infinite transmission coefficient, τ_∞:

$$\tau_F = \frac{I_r^F}{I_0}\frac{I_r^{\infty}}{I_r^{\infty}} = \tau_\infty \frac{I_r^F}{I_r^{\infty}} \tag{21}$$

The insertion loss of the layer is assumed independent of the aperture.

The ratio $\frac{I_r^F}{I_r^{\infty}}$ is equal to the ratio of the intensity radiated by the velocity trace of the incident pressure to the intensity radiated by the infinite aperture. In next formulas emitter and receiver fluids are assumed of different nature.

$$I_r^F = \sigma_{rad}\rho_r c_r v^2{}_{n0} = \sigma_{rad}\frac{\rho_r c_r p_0^2}{\left[\rho_0 c_0\right]^2}\cos^2\theta_0 \text{ and } I_r^{\infty} = \frac{p_0^2}{\rho_0 c_0}\cos\ \theta_r \tag{22}$$

$$\frac{I_r^F}{I_r^{\infty}} = \sigma_{rad}\frac{\rho_r c_r}{\rho_0 c_0}\frac{\cos^2\theta_0}{\cos\theta_r} \tag{23}$$

For identical fluids both sides of the aperture, the ratio of finite to infinite intensity becomes:

$$\frac{I_r^F}{I_r^{\infty}} = \sigma_{rad}\cos\theta_0 \tag{24}$$

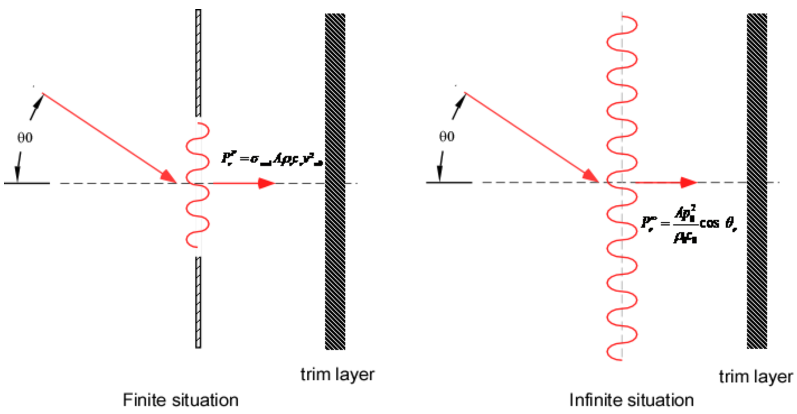

Figure 3. Radiation efficiency of the aperture.

Formula (24) proposes a cosinus/radiation efficiency weighting of the infinite transmission loss and requires to calculate the radiation efficiency of the aperture prior to apply the correction. This correction is called ***Sigmarad*** correction. Advanced demonstration of this correction may also be found in [4]. Formula (14) may be modified using the normal velocity to the aperture, v_n. In that case, (14) becomes:

$$P_{rad}^{F}(\omega,\theta)=\frac{1}{2\pi}\int\frac{\rho_f c_f}{\cos\theta}\left|\tilde{v}_n(k_y)\right|^2 dk_y=\frac{k_0}{2\pi}\int\frac{\rho_f c_f}{\sqrt{k_y^2-k_0^2}}\left|\tilde{v}_n(k_y)\right|^2 dk_y \tag{25}$$

The operator $1/\sqrt{k_y^2-k_0^2}$ is proportional to the infinite radiation efficiency of the aperture and (14) may be then re-formulated as (22). The simple convolution correction does not operate on normal velocity and will imply some limitation in low frequency range as shown in next numerical examples.

4 NUMERICAL EXAMPLES

4.1 Mass layer transmission loss

The Transmission Loss in dB is computed as $TL=-10\log\left(\frac{I_t}{I_{inc}}\right)$ in SEA+ software [5]. SEA+ implements both ***Convolution*** and ***Sigmarad*** correction for transmission loss calculation using TMM. In that case graphed *TL's* are all random-incidence *TL* obtained by averaging discrete incident $TL(\theta)$ (over 0 to 90°). Fluid is standard air both sides.

TL is predicted for a 2.7 mm-thick mass layer with volume density 1000 kg/m^3. The mass layer has then a density of 2.7 kg/m². The area is fixed to 1x1 m².

Predicted TL's are given in Figure 4 for three configurations: no correction, ***Convolution*** correction and ***Sigmarad*** correction. In Figure 4-left, spatial windowing increases the TL of 2 dB on average around 1000 Hz and below, larger correction factor is obtained with ***Sigmarad*** below 300 Hz (up to + 5 dB). In Figure 4 right, the opening is 5x5 m². The spatial windowing correction is found smaller as expected.

4.2 Dash panel firewall insulation

The firewall of a dash panel may incorporate many layers as shown hereafter in Figure 5 (left). In that particular case, the spatial window is about same effect than in previous case :+ 5 dB correction for ***Sigmarad*** correction in figure 5 (right).

4.3 Mass layer transmission loss in water

The calculation of transmission loss is performed by inserting between two SEA water cavities a 1x1 m² mass layer of mass density 390 kg/m². Figure 6 shows the effect of spatial windowing on TL leading to an increase of around 4 dB at low frequencies.

4.4 Spatial windowing of mechanical line junctions

The convolution correction is used also for correcting analytical calculation of line junctions SEA coupling loss factors (CLF) in [5]. This correction is applicable when two plates with a common edge are only physically connected on a fraction of this edge.

Figure 7 shows the predicted configuration. The Coupling Loss Factor (CLF) is first identified numerically by the Virtual SEA (VSEA) method [2], [6], applied to FEM model of the two coupled plates. This model takes into account the actual coupling geometry: the two plates are welded on 100 mm on a fraction of plate edges (1m length). VSEA model delivers the flexural CLF between the two plates in direction 1-2 and 2-1. Plate 1 is made of 1-mm steel and plate 2 of wood (10 mm-

OSB). Both systems are meshed with plate elements in Nastran and modal extraction is pushed up to 2000 Hz which corresponds to the frequency band of VSEA CLF identification.
Second, CLF is predicted from two coupled analytical plates of which transmission loss is computed from wave theory in SEA+. In the analytical model, plates are connected on a line of 100 mm. The edge length is declared equal to 1 m. Spatial windowing correction is successively activated and deactivated and resulting CLF's are compared with VSEA CLF. Results are given in Figure 8. We can see the effect is much more important than in previous acoustic cases. The spatial windowing analytical prediction is closer from VSEA CLF. Spatial windowing is thus a useful tool to improve the predictive quality of analytical mechanical transmission in SEA models.

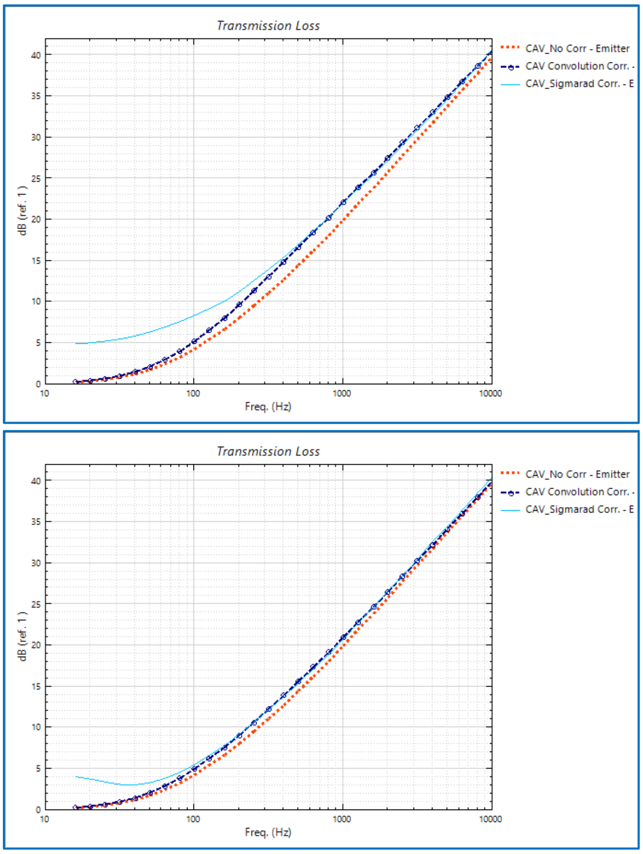

Figure 4. TL with spatial windowing for 2.7 kg/m² mass layer Area 1x1m² (left) & 5x5 m² (right).

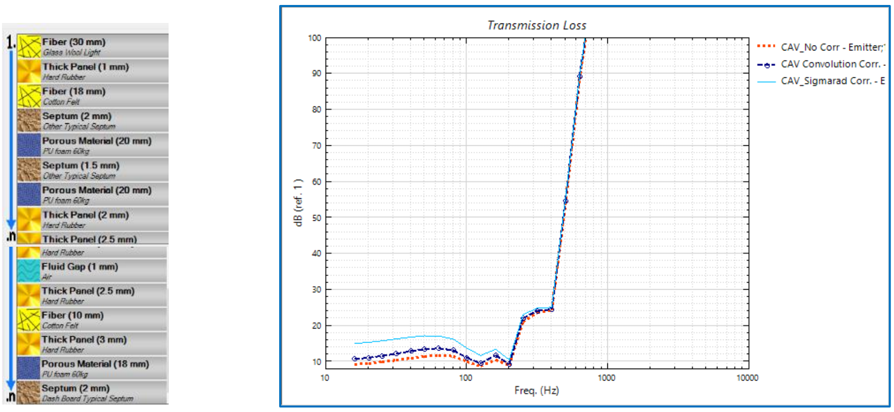

Figure 5. On right TL of dash panel (firewall) with and without spatial windowing (Area 1x1.5 m²) and on left dash panel layers.

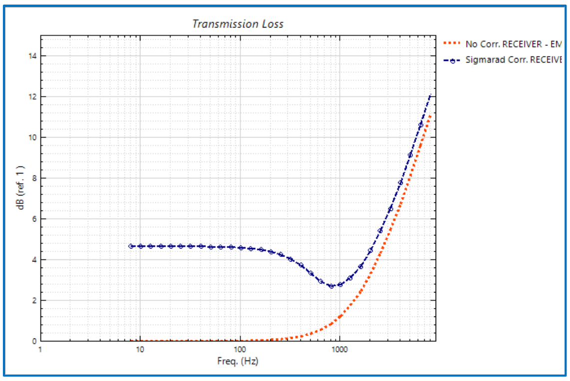

Figure 6. TL of mass layer 390 kg/m^2 between two sea water cavities (without spatial windowing and with ***Sigmarad*** correction).

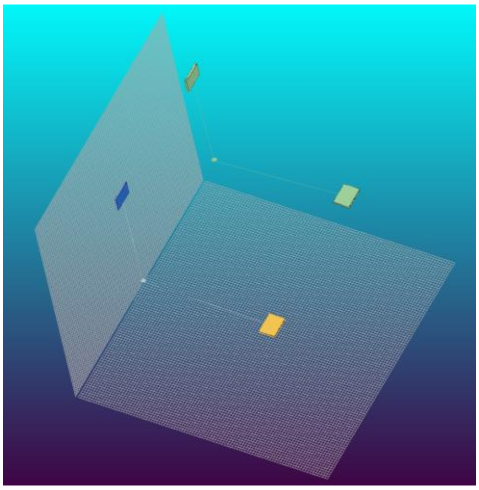

Figure 7. Spatial windowing correction of structural line-junction CLF (two square-1 m² plates connected at right angle on 100 mm along 1 m edge).

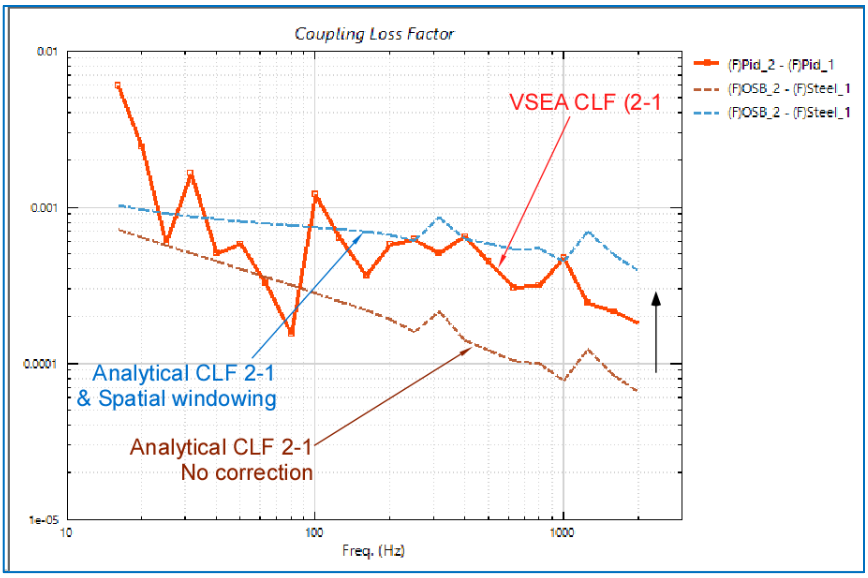

Figure 8. Spatial windowing correction of structural CLF compared to VSEA CLF.

5 CONCLUDING REMARKS

Spatial windowing technique is an efficient way to make analytical predictions closer to measured data. The amplitude of the correction depends on the physics of the model. We have seen an attenuation of the mass-driven sound transmission of around 4 to 5 dB in the low frequency regime for the acoustic coupling. For structural coupling, corrections vs. frequency are more broadband and may lead to increase of vibration transmission. They should be enabled when only a fraction of connected plate edge is allowed to transmit noise. More generally, the correction is valid when the aperture size of the junction is smaller than the container (plane or line) size. For dynamical systems with complex topology modelled using SEA, some care has then to be taken before to apply the relevant corrections.

REFERENCES

[1] L. Beranek and L. Istvan, *Noise and Vibration Control Engineering: Principles and Applications*, Second Edition, Wiley.

[2] Gagliardini L., Houillon L., Petrinelli L. and Borello G., "Virtual SEA: Mid-Frequency Structure-Borne Noise Modeling Based on Finite Element Analysis", 03NVC-94, SAE 2003.

[3] Villot M., Guigou-Carter C. and Gagliardini L., Predicting the Acoustical Radiation of Finite Size Multi-Layered Structures by Applying Spatial Windowing on Infinite Structures, *Journal of Sound and Vibration*, 245(3), 433-455, 2001.

[4] Allard J. F. and Atalla N., *Propagation of Sound in Porous Media*, Wiley, 2009.

[5] Borello G., *SEA+ Advanced Theory* manual 2016, InterAC.

[6] Borello G., *SEAVirt Theory* manual 2016, SEA+ software module, InterAC.