PREDICTIVE CAPABILITIES OF FOUR FINITE STRAIN VISCOELASTIC MODELS UNDER SEPERABILITY ASSUMPTION

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ABSTRACT

The predictive capabilities of some integral-based finite strain viscoelastic models under the time-strain seperability assumption have been investigated through experimental data for monotonic, relaxation and dynamic shear loads, in time and frequency domains. This survey is instigated by experimental observations on three vulcanized rubber material intended for an engineering damping application. Models under consideration are Christensen, Fosdick & Yu, a variant of BKZ model and the Simo model. In the time domain, for each test case and for each model, the nominal stress is hence compared to experimental data, and the predictive capabilities are then examined with respect to three polynomial forms hyperelastic potentials. In the frequency domain, the predictive capabilities have been analysed with respect to the frequency and predeformation effects.

1 INTRODUCTION

Elastomeric compounds are widely used in industry for their mechanical properties particularly their damping capabilities e.g tires, shock-absorbing bushes, construction industry, aerospace applications... To design industrial compounds efficiently, it is of major importance to be able to predict the impact of the nonlinearity effects on the products, and estimating the damping capability is a primary feature to be considered in many engineering applications. While many contributions investigated either the purely elastic phenomena for elastomers at large deformations [1] or the viscoelastic phenomenon [2], the attention is here focused on the hysteritic time dependent part of the response.

The objective of the current work is the analysis of the predictive capabilities of some heriditary integral-based constitutive models in time and frequency domains, under the separability assumption [3][4]. From an historically point of view, the constitutive theory of finite linear viscoelasticity [5] have been of a major contribution and is founded on an extension of the Boltzmann superposition principle to finite strain. The stress quantity is decomposed to an equilibrium part corresponding to the stress response at highly slow rate, and an overstress quantity expressed as an heriditary integral including a measure of material's memory through relaxation functions. Based on experimental observations, the time-strain separability or factorability assumption [4] is frequently introduced in the formulation of finite strain viscoelasticity constitutive models and afford a large theoretical simplicity.

2 MODELS UNDER CONSIDERATION

In the present work, some of major contributions finite strain viscoelastic models involving heriditary integral have been considered under the seperability assumption, chosen so as to not require a special identification procedure. All parameters have been identified using Abaqus Evaluate Module. The models under consideration are: Christensen [6], Fosdick & Yu [7], a variant of BKZ [8] and Simo Model [9]:

$$\boldsymbol{\sigma}^{Ch} = -p\mathbf{I} + 2\mathbf{B}\frac{\partial W}{\partial \mathbf{B}} + \mathbf{F}\,G_0\int_0^t g_1(t-s)\frac{\partial \mathbf{E}(\mathbf{s})}{\partial s}ds\mathbf{F}^T \tag{1a}$$

$$\boldsymbol{\sigma}^{FY} = -p\mathbf{I} + 2\mathbf{B}\frac{\partial W}{\partial \mathbf{B}} + G_0\int_0^t g_1(t-s)\frac{\partial \mathbf{E}_t(s)}{\partial s}ds \tag{1b}$$

$$\boldsymbol{\sigma}^{BKZ} = -p\mathbf{I} + 2\mathbf{B}\frac{\partial W}{\partial \mathbf{B}} - 2\,\mathbf{F}\,G_0\int_0^t g_1(t-s)\frac{\partial \mathbf{C}^{-1}}{\partial s}ds\,\mathbf{F}^T \tag{1c}$$

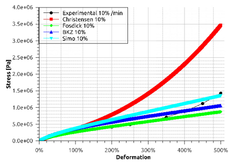
$$\begin{aligned}\boldsymbol{\sigma}^{Si} = & -p\mathbf{I} + 2\mathbf{B}\frac{\partial W}{\partial \mathbf{B}}\frac{1}{g_\infty} + \\ & dev\left[\int_0^t \frac{\partial g_1(s)}{\partial s}\mathbf{F}_t^{-1}(t-s)\frac{2}{g_\infty}\mathbf{B}(t-s)\frac{\partial W}{\partial \mathbf{B}}\mathbf{F}_t^{-T}(t-s)\,ds\right]\end{aligned} \tag{1d}$$

where $\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}$ is the deformation gradient. The right and left Cauchy-Green strain tensor are consecutively $\mathbf{C} = \mathbf{F}^T\mathbf{F}$ and $\mathbf{B} = \mathbf{F}\mathbf{F}^T$. The Green-Saint-Venant strain tensor is $\mathbf{E} = \frac{1}{2}(\mathbf{C} - \mathbf{I})$. the hyperelastic free energy potential $W = W(I_1, I_2)$ and I_1 and I_2 stands for the isotropic scalar-valued invariants of $\mathbf{C}$. $g_1(t)$ is the dimensionless relaxation kernel defined as a Prony series and commonly taken as: $g_1(t) = \sum_{i=1}^N g_i(e^{\frac{-t}{\tau_i}})$ with g_i and τ_i are material's parameters. $g_i > 0$, $g_\infty = 1 - \sum_{i=1}^N g_i$. G_0 is the instantaneous linear shear modulus.

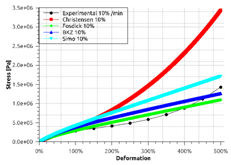
3 ON THE CAPABILITY TO PREDICT TIME-DEPENDENT EXPERIMENTS

3.1 Monotonic tests

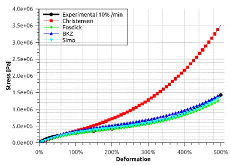
The available experimental data are for an uniaxial tension test and a simple shear test, with different strain-rates. Considering purely hyperelastic response, we make use of the equilibrium strain-stress curves for the identification of the hyperelastic potential. Herein, we made the choice on the polynomial hyperelasic form and its particular cases NeoHookean and Mooney-Rivlin. Considering viscoelastic phenomena, we identified the prony series through normalized shear relaxation data.
The response of monotonic tension/shear nominal stress for bromobutyl BIIR material are reported in Fig.1 . The considered models present the capability to take into account a strain rate effect, with higher stain rates leading to a higher stress at same deformation level. Considering a Neo-Hookean or a Mooney-Rivlin hyperelastic potential, the predicted data are seen to be non accurate, and all the models could not predict the second inflection point. Considering the 2^{nd} Order Polynomial hyperelastic potential, we observed that the Christensen model (1a) is seen to highly overestimate the nominal stress level for high strains, not to exceed 100% of deformation. Fosdick & Yu model (1b) is seen to underestimate the stress level for the three materials, and has the lowest stress level through all models. Nevertheless, the predicted level is seen to be acceptable. Meanwhile, both BKZ (1c) and Simo (1d) models were able to give a better approximation of the stress level. The prediction is quite good and the predicted stress is in a good range.



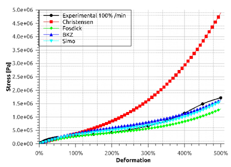
(a) NeoHoohean 10 % $\text{min}^{-}1$



(b) Mooney Rivlin 10 % $\text{min}^{-}1$



(c) 2^{nd} Ord. poly 10 % $\text{min}^{-}1$



(d) 2^{nd} Ord. poly 100 % $\text{min}^{-}1$

Figure 1: BIIR monotonic tension

3.2 Relaxation tests

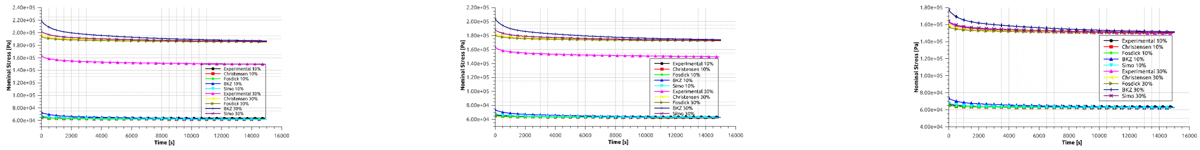
The evaluation of the prony series is available in the abaqus evaluation module for normalized shear stress relaxation experiments. The deformation taken into account for shear relaxation tests is less than 50% of deformation. For a very long relaxation time i.e $t \to \infty$, the relaxation equilibrium expression:

$$\sigma_{12}^{Poly\,Equil} = 2\left(C_{10} + 2C_{20}\gamma_0^2 + C_{11}\gamma_0^2 + C_{01} + 2C_{02}\gamma_0^2 + C_{11}\gamma_0^2\right)\gamma_0 \quad (2)$$

Comparison of models response is graphically shown in Fig. 2. We observed that the Neo-Hookean hyperelastic potential, as well as Mooney-Rivlin, the models are seen to not well predict the relaxation test data. The 2^{nd} order Polynomial hyperelastic model offers the best prediction for the long-term relaxation stress response and the measured error is of an acceptable level. The major difference between models is seen for the hysteritic part. The Simo model is seen to offer a good fidelity to approximate low times stress. Christensen and Fosdick & Yu models underestimate the hysteritic stress level while the BKZ model is observed to highly overestimate the instantaneous relaxation stress.

4 ON THE CAPABILITY TO PREDICT FREQUENCY-DEPENDENT EXPERIMENTS

The determination of the complex shear modulus was introduced by [6] and is a Fourrier transform of the governing equations defined for a kinematically small perturbation about a pre-deformed state. Since the available experimental data in the frequency domain are limited to



(a) NeoHookean (b) Mooney-Rivlin (c) 2^{nd} Order Polynomial

Figure 2: Relaxation response with different hyperelastic model: Material NR

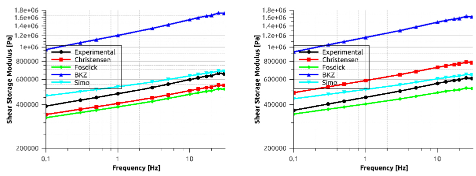
30%, and the procedure is linearized for high order strains, a Mooney-Rivlin potential leads to sufficient results. Therefore, we used the following state of loading:

$$\gamma(s) = 0 \quad s < 0 \quad ; \quad \gamma(s) = \gamma_0 \quad 0 \leqslant s \leqslant t_0 \quad ; \quad \gamma(s) = \gamma_0 + \gamma_a e^{(i\omega t)} \quad t_0 \leqslant s \leqslant t \tag{3}$$

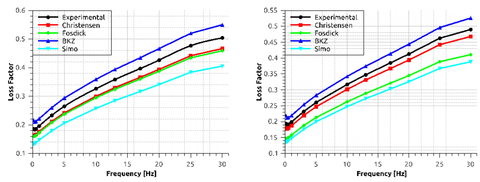
We assume that $|\gamma_a| << 1$ and that a steady-state solution is obtained. The dynamic stress has the form:

$$\sigma^*(\omega) = G^*(\omega, \gamma_0)\gamma(\omega) \quad ; \quad G^*(\omega, \gamma_0) = G_s(\omega, \gamma_0) + iG_l(\omega, \gamma_0) \tag{4}$$

where $G_s = \Re\left[G^*(\omega, \gamma_0)\right]$ and $G_l = \Im\left[G^*(\omega, \gamma_0)\right]$ are the shear storage and loss modulus. As shows Fig 3, following observations have been made for the shear storage modulus: Simo model have shown an excellent approximation of the dynamic shear storage modulus with respect to frequency and predeformationw while Christensen model underestimates the shear modulus at 10% of deformations and overestimate the properties at higher predeformation: this model was not able to predict the softening of the material occuring with increasing predeformation level. Fosdick and Yu model's response underestimates the materials response and the BKZ model's response is not in an acceptable range. Interested in the shear loss factor, the frequency dependence of the compared models is pronounced, and all models are seen to offer a good approximation of this factor. The Simo model slightly underestimate the response, and the maximum deviation is of about 10%. One can observe that although the BKZ model could not predict the storage modulus, is have shown the ability to well approximate the damping of the materials.



(a) shear storage, 10% predeformation (b) shear storage, 30% predeformation



(c) loss factor, 10% predeformation (d) loss factor, 30% predeformation

Figure 3: NR/BIIR dynamic properties at different predeformation levels

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