A SPECTRAL BOUNDARY ELEMENT APPROACH TO REPRESENT SCATTERED WAVES IN UNBOUNDED ACOUSTIC REGIONS

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ABSTRACT

This work presents a two-and-a-half (2.5D) spectral formulation based on the boundary element method (BEM) to study three dimensional (3D) wave propagation within acoustic regions. The BEM is used to analyse the acoustic field in unbounded regions with rigid cavities with arbitrary cross-section. The BEM is extended to its spectral formulation using Lagrange interpolant polynomials as element shape functions at the Legendre-Gauss-Lobatto (LGL) points. The proposed method is verified from a benchmark problem regarding the acoustic scattered wave in an unbounded medium by a rigid cavity. A h-p analysis is carried out to assess the accuracy of the method. The results show a high accuracy of the proposed method to represent this kind of problem.

1 INTRODUCTION

Time-harmonic wave propagation, such as fluid acoustics and solid scattering, is a common phenomenon that appears in many engineering fields. The propagation of acoustic waves triggered by static and moving pressure sources, the vibration assessment and the acoustic insulation involve fluid and solid interaction and must be considered rigorously. The finite element method (FEM) have been used is several works to predict the response in fluid-structure interaction problems. For the low frequency range, the conventional finite elements with linear shape represent accurately the fluid and solid scattering waves. However, at high frequencies, these shape functions do not provide reliable results due to so-called pollution effects [1, 2]: the accuracy of the numerical solution deteriorates with increasing non-dimensional wave number and it is not sufficient the commonly employed rules of n elements per wavelength [3]. High element resolutions are required in order to obtain results with reasonable accuracy.

The method proposed in this work regards with a two-and-a-half dimensional (2.5D) approach to represent scattered waves in fluid media. The proposed approach is useful for problems where the material and geometric properties are uniform along one direction, and the source exhibits 3D behaviour.

2 NUMERICAL MODEL

The 2.5D formulation computes the problem solution as the superposition of two-dimensional (2D) problems with a different longitudinal wavenumber, k_z, in the z direction. An inverse Fourier transform is used to compute the 3D solution:

$$a(\mathbf{x},\omega) = \int_{-\infty}^{+\infty} \widehat{a}(\widehat{\mathbf{x}}, k_z, \omega) e^{-ik_z z} dk_z \tag{1}$$

where $a(\mathbf{x},\omega)$ is an unknown variable (e.g., displacement or pressure), $\widehat{a}(\widehat{\mathbf{x}}, k_z, \omega)$ is its representation in the frequency-wavenumber domain, $\widehat{\mathbf{x}} = \mathbf{x}(x, y, 0)$, ω is the angular frequency, and $i = \sqrt{-1}$.

2.1 The 2.5D spectral boundary element formulation

The boundary element formulation presented in this work considers an arbitrary boundary submerged in an unbounded fluid medium. The integral representation of the pressure p^i for a point i located at the fluid subdomain $\Omega_{f\infty}$, with zero body forces and zero initial conditions may be written as [4]:

$$c^i p^i(\mathbf{x}^i,\omega) = \int_{\Gamma_f} p^{i*}(\mathbf{x},\omega;\mathbf{x}^i) u^i(\mathbf{x},\omega) d\Gamma - \int_{\Gamma_f} u^{i*}(\mathbf{x},\omega;\mathbf{x}^i) p^i(\mathbf{x},\omega) d\Gamma \tag{2}$$

where $u^i(\mathbf{x},\omega)$ and $p^i(\mathbf{x},\omega)$ are respectively the normal displacement to boundary Γ_f and the nodal pressure. $u^{i*}(\mathbf{x},\omega;\mathbf{x}^i)$ and $p^{i*}(\mathbf{x},\omega,\mathbf{x}^i)$ are respectively the fluid full-space fundamental solution for normal displacement and pressure at point $\mathbf{x}$ due to a point load at $\mathbf{x}^i$. The integral-free term c^i depends only on the boundary geometry at point i. The integration boundary Γ_f represents the boundary between the unbounded fluid medium ($\Omega_{f\infty}$) and the solid subdomain (Ω_s). The proposed spectral boundary element method for the 2.5D fluid element uses Legendre polynomials of order p as interpolation shape functions, where the local nodal coordinates ξ are found at the LGL integration points.

3 NUMERICAL VERIFICATION

The BEM model was verified with a benchmark problem. The model was implemented and validated by applying it to a fixed cylindrical circular cavity, submerged in a homogeneous unboundend fluid medium. The cavity is subjected to a harmonic point pressure load. The analytical solution to this problem can be found in Reference [5].

The cavity had a radius $r = 5$ m , located at the origin $(x, y) = (0, 0)$. The unbounded fluid medium properties were pressure wave velocity $\alpha = 1500$ m/s and density $\rho = 1000\,\mathrm{kg/m^3}$. The problem solution was computed for a dilatational point source placed at the fluid medium $\widehat{\mathbf{x}}_0 = (x_0, y_0) = (0, 15)$ 15 m away from the cavity centre. This loads emits a harmonic incident field $\widehat{p}_{inc}$ at a point $\widehat{\mathbf{x}}$ described by:

$$\widehat{p}_{inc} = (\widehat{\mathbf{x}}, \omega, k_z) = \frac{-iA}{2} H_0^{(2)}(k_\alpha \sqrt{(x - x_0)^2 + (y - y_0)^2}) \quad (3)$$

where A is the source amplitude, $H_0^{(2)}$ is the Hankel function of the second kind, and $k_\alpha = \sqrt{\alpha/\omega}$. In this problem, the longitudinal wavenumber was set to $k_z = 0$.

The problem solution was computed over a grid of 1376 receivers regularly spaced in a outer region defined by $-10\mathrm{m} \leq x \leq 10\mathrm{m}$ and $-10\mathrm{m} \leq y \leq 10\mathrm{m}$. Figure 1 shows the convergence curves for the scaled L_2 error ϵ_2. The curves show a monotonic convergence with the element order p. Finest meshes tend to a minimum error with a lower element order p.

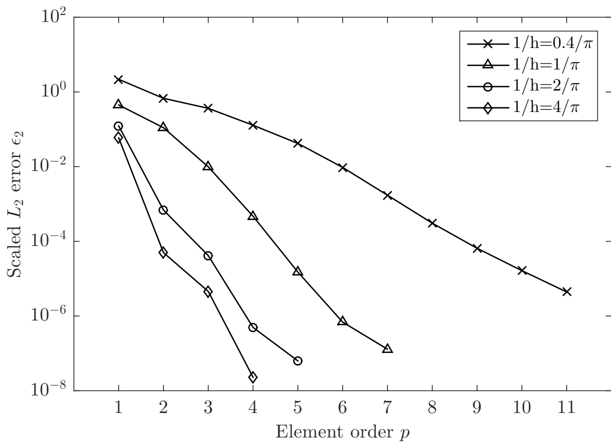

Figure 1: Convergence of scaled L_2 error ϵ_2 for different discretisations $1/h$ and element polynomial orders p.

4 CONCLUSIONS

This work has proposed a spectral based formulation based on the BEM to study acoustic wave propagation. This method looks at 3D problems whose materials and geometric properties remain homogeneous in one direction. A spectral 2.5D approach for fluid-acoustics media was developed to avoid the pollution effect at high frequencies in the problem solution. The model was verified with a benchmark problem with known analytical solution. The numerical result was in good agreement with the reference solution.

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