UNCERTAINTY QUANTIFICATION IN MID-FREQUENCY RANGE SIMULATIONS USING THE STATISTICAL MODAL ENERGY DISTRIBUTION ANALYSIS

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ABSTRACT

The Statistical Modal Energy Analysis (SmEdA) is a variant of the Statistical Energy Analysis (SEA) developed to predict the high frequency behaviour of structures by dividing them into subsystems without requiring a modal energy equipartition. The method is based on the modal bases of uncoupled subsystems, and coupling loss factors are derived from Finite Element Analysis. Uncertainty Quantification can thus be applied in such a configuration at either the subsystem level, with respect to the physical input parameters (eg material properties and dimensions), or at the coupled model level with respect to the coupling factors or the modal data used to compute them. For UQ to be physically meaningful, it is necessary that uncertainty modeling at the coupled model level be representative of uncertainty at the subsystem level. A strategy based on sampling at the coupled model level using a covariance matrix computed at the subsystem level is proposed here. The methodology is formulated and applied to a four-subsystem structure. The UQ performed at the two levels is shown to be coherent but with reduced computational costs at the coupled model level allowing a higher number of UQ simulations.

1 INTRODUCTION

The prediction of high frequency noise and vibration levels requires the use of specific methods. One well-known approach is the Statistical Energy Analysis (SEA) that provides a statistical average of the vibratory or acoustical behavior of the structure of interest [1, 2]. The full system is divided into subsystems and energy flows between these subsystems are computed. The parameters and equations are obtained under certain hypotheses [3], one of which is the modal equipartition of energy in subsystems. To overcome this limitation, the SmEdA approach has been developed as a reformulation of SEA without requiring energy equipartition [4]. The modal energy equations lead to coupling coefficients derived from finite element simulations.
In an uncertain context the evaluation of the impact of different sources of uncertainty on the output quantifies of interest, called Uncertainty Quantification (UQ), is an important part of a global Quantification of Margins and Uncertainties (QMU). The effect of uncertainties in SEA models has been studied using many approaches such as the partial derivative analysis and the design of experiments on SEA factors [5], parametric methods on FE components and non-parametric studies on SEA elements in a hybrid FEM/SEA approach [6]. It thus appears that uncertainty quantification can be performed using either the input physical parameters or the coupled parameters involved in the energy equations and derived from the physical inputs [7].
UQ at the coupled model level is computationally more efficient but has no direct physical meaning. It is thus necessary to include information about how uncertainty is propagated from the subsystem level. A strategy based on the use at the coupled model level of a covariance matrix computed at the subsystem level is proposed here. The SmEdA equations are first recalled to introduce the coupled parameters, then the proposed methodologyis presented, and finally this methodology is applied to an academic model of four coupled plates.

2 SMEDA EQUATIONS

SmEdA relies on the equations of the SEA. The main difference between both approaches is that SmEdA not only describes the coupling between subsystems but also the coupling between the individual modes of the different subsystems. In this way the restrictive modal equipartition assumption is not required. The resulting formulation for a two-subsystems model can be written as follows,

$$\Pi_{inj}^{p} = \left(\omega_p \eta_p + \sum_{q=1}^{M_2} \omega_c \eta_{pq}\right) E_p - \sum_{q=1}^{M_2} \omega_c \eta_{pq} E_q, \ \forall p \in [1, ..., M_1], \tag{1}$$

$$\Pi_{inj}^{q} = - \sum_{p=1}^{M_1} \omega_c \eta_{pq} E_p + \left(\omega_q \eta_q + \sum_{p=1}^{M_1} \omega_c \eta_{pq}\right) E_q, \ \forall q \in [1, ..., M_2]. \tag{2}$$

where p and q are modes of subsystems 1 and 2, respectively, with corresponding natural frequencies, ω_p and ω_q, M_1 and M_2 are the number of modes for subsystems 1 and 2. The internal loss factors (ILF) η_p and η_q are computed for each mode, and the coupling loss factors (CLF) η_{pq} are derived for a pair of modes rather than for a subsystem pair. The subsystem energies are then determined as sums of the modal energies,

$$E_1 = \sum_{p=1}^{M_1} E_p, \quad E_2 = \sum_{q=1}^{M_2} E_q. \tag{3}$$

The method requires calculating the modes of each uncoupled subsystem, generally determined from a Finite Element (FE) analysis, and it can be considered as an approach for which uncertainty quantification can be performed at different levels: the first one, at the subsystem level, where uncertainty is introduced in the input parameters (material, geometry), and the second one, at the coupled model level where uncertainty is introducing in the modal data.

3 METHODOLOGY

The figure 1 presents the flowchart of activities performed to quantify uncertainties at the subsystem and at the coupled model levels. Mesh refinement ensures the insensitivity of results to mesh discretization and a significant level of prediction uncertainty. Sampling is performed with the Monte Carlo (MC) method based on an assumed multivariate normal distribution. This sampling uses the mean and covariance matrix that define the distribution: the covariance matrix informs the sampling at the coupled model level using the results of sampling at the subsystem level. Effect screening is useful to limit uncertainty quantification to the most influential variables leading to significant computational savings.

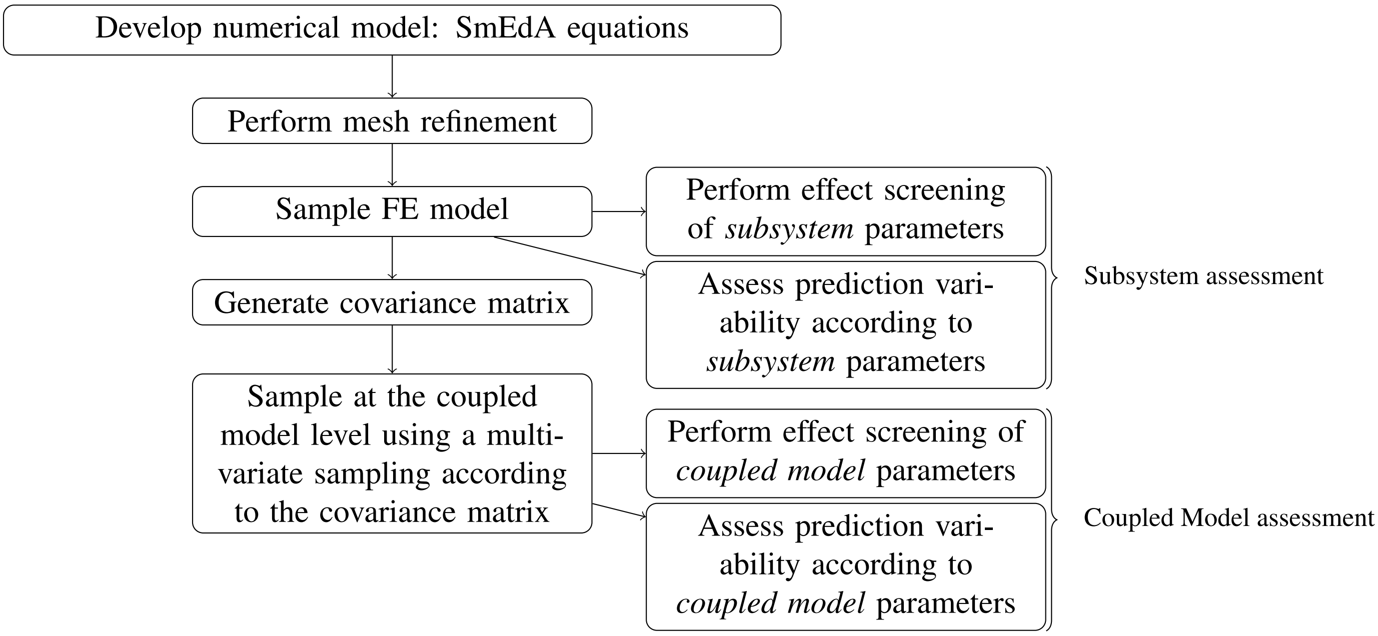


Figure 1: Uncertainty Quantification at the *subsystem* and at the *coupled model* levels.

4 CASE-STUDY APPLICATION

The methodology is applied to a four-subsystem model shown in figure 2. The plates are made of steel ($E = 210$ GPa, $\rho = 7800$ kg.m^3), with a constant damping ratio of 0.05 for all subsystems. Mesh refinement is performed both at the subsystem level and at the coupled system level to ensure less than 1% error on the natural frequencies up to 2 kHz, and the convergence of the number of modes and SmEdA energies. 5000 Monte Carlo samples are performed at the coupled model level and the vibratory energies thus obtained are compared to those resulting from 100 Monte Carlo samples performed at the subsystem level.

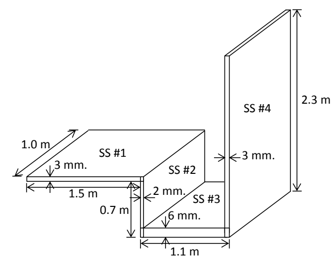
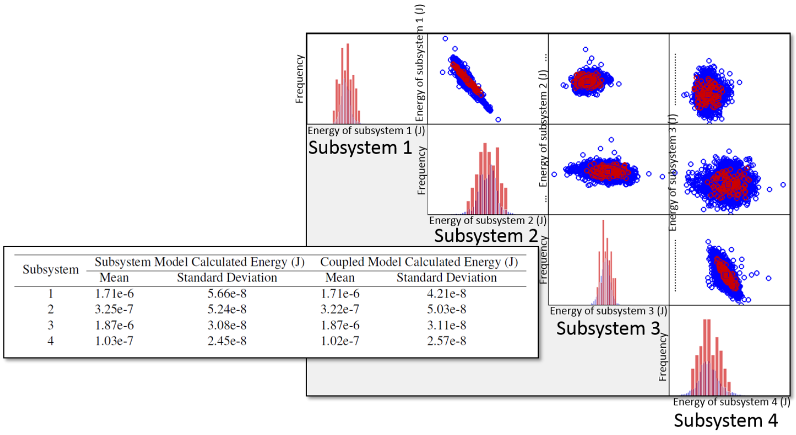


Figure 2: Four-subsystem model.

Figure 3 presents the vibratory energies obtained at the subsystem level (in red) and at the coupled level (in blue), and the global statistics for the Monte Carlo sampling : the diagonal plots show the comparison of the SmEdA histograms for each subsystem, the out-diagonal plots show the comparison of the output-output scatter plots for pairs of subsystems. The consistency in the obtained results demonstates that the use of coupled data such as the natural frequencies constitute an adequate proxy to the use of the input parameters for uncertainty quantification, leading to a significant reduction in computational costs.



Subsystem	Subsystem Model Calculated Energy (J)		Coupled Model Calculated Energy (J)	
	Mean	Standard Deviation	Mean	Standard Deviation
1	1.71e-6	5.66e-8	1.71e-6	4.21e-8
2	3.25e-7	5.24e-8	3.22e-7	5.03e-8
3	1.87e-6	3.08e-8	1.87e-6	3.11e-8
4	1.03e-7	2.45e-8	1.02e-7	2.57e-8

Figure 3: Comparison of results obtained from MC sampling at the *subsystem* and *coupled model* levels.

5 CONCLUDING REMARKS

UQ at the subsystem level leads to results that are physically meaningful but computationally expensive, while UQ at the coupled model level is harder to interpret but computationally more efficient. The proposed approach uses a covariance matrix informed by minimal sampling at the subsystem level to propagate uncertainty at the coupled system level. The results are shown to be consistent and the reduction in computational burden allows to increase the range of predictions.

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