THE INDUCED BY MESHING STIFFNESS VARIATION DYNAMICS OF PLANETARY GEARS USING AN ITERATIVE SPECTRAL METHOD.

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ABSTRACT

We investigate in this study the dynamic behavior of planetary gear systems induced by the time-varying mesh stiffnesses of the sun-planet and the ring-planet meshes. These internal excitations are of parametric kind and induce, under stationary conditions, multi-frequencial responses. These responses lead to parametric resonances when natural eigenmodes defined from the mean values stiffnesses are excited. In order to compute the planetary gear dynamic responses, a previously defined iterative spectral method is introduced and extended to this context, significantly saving computation times. By expanding the solution in the modal basis computed from the mean characteristics of the system, this method is derived in the frequency domain and directly provides the spectrum of the response.
As an example, a simple single stage planetary gears multi-degree-of-freedom modelling is built. Eigenmodes, dynamic transmission errors and dynamic mesh forces are computed and analyzed. Finally, comparisons with the Runge-Kutta time integration scheme is performed to demonstrate the method validity.

1 INTRODUCTION

Planetary gear systems form a reliable and efficient way to design compact power transmissions with high gear ratio. Consequently, these transmissions are very popular from wind turbines to home automation applications, or again automatic gearboxes, to only cite few cases. Concerning the NVH performance, their noise characteristics remain often unacceptable, especially in view of the more restrictive standards in the context of reduction in pollutant and greenhouse gas emissions. In particular, demand is now strong for reducing weight while respecting the vibroacoustic performance and reliability. In this context, it becomes of great importance to introduce advanced simulation tools applying right to the design stage. Whining noise of planetary gears is especially addressed in this study. This dominant noise results from the dynamic behaviour of the gears induced by the static transmission errors. These main excitation sources at each meshing result from the teeth deflection and manufacturing errors. In the context of dynamic modelling, they are introduced as displacement periodic excitations and internal parametric excitations (meshing stiffness fluctuations). Particularity of the planetary gears, these last induce couplings between the multiple meshings. Consequently, equations of motion projected into the modal base built from the time averaged mesh stiffness remains coupled. The principal aim of this study is to demonstrate the capability of a previously introduced iterative spectral method [1-2] to treat the special case of planetary gears. This method is devoted to compute the stationary dynamic responses of parametrically excited systems with time-varying characteristics, such as stiffness, subjected to stationary deterministic or stochastic external excitation. The case of standard cylindrical gears are well mastered with this method compared to experimental results [3-4].

2 DYNAMIC MODELLING

Consider a single stage planetary gear, we introduced a lumped dynamical multi-degree-of-freedom model similar to that introduced in [5]. This planetary gear is composed of 3 planets. By neglecting the centrigugal and Coriolis forces, the matrix equation of motion is given by

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \sum_{j=1}^{6} k_j(t)\mathbf{R}_j\mathbf{R}_j^t\mathbf{x} = \sum_{j=1}^{6} k_j(t)\mathbf{R}_j\mathbf{R}_j^t\mathbf{x}_s \tag{1}$$

In this equation $\mathbf{M}$ and $\mathbf{C}$ are respectively the mass and damping matrices, $\mathbf{R}_j$ is a structural vector witch couples wheels through the j-th meshing process, $\mathbf{x}$ is the dynamic response co-ordinates vector, $\mathbf{x}_s$ is the static ones, and $k_j(t)$ is the time-varying meshing stiffness of the j-th mesh which represents the internal parametric excitation. Summation over 6 corresponds to the 3 sun-planet meshes and the 3 ring-planet meshes.

By considering the free undamped time-averaged system, we can defined a mean modal basis, which appears to be the pertinent basis for describing the parametric resonance phenomena. This modal basis is then deduced from the following eigenvalue problem

$$\left(\sum_{j=1}^{6} \overline{k_j}\mathbf{R}_j\mathbf{R}_j^t - \Omega^2\mathbf{M} \right)\mathbf{V} = \mathbf{0} \tag{2}$$

where $\overline{k_j}$ represents the mean time value of $k_j(t)$. As an example, a typical mode is shown in Figure 1. In practice, it is possible also to distinguish the nature of these modes by meshing storage energy computation.

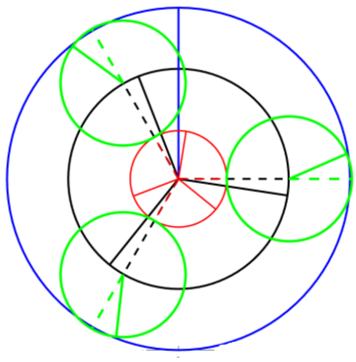

Figure 1. Typical mode with the modal motion of the planets and the sun.

3 COMPUTATIONAL PROCEDURE FOR THE STATIONARY RESPONSES

In order to simulate and compute the stationary coupled dynamic responses of the transmission model (which are multifrequential), we introduce the iterative spectral method. This method is based on the direct computation of the solutions in the spectral domain. To this end the matrix equation (1) is rewritten in the modal base as

$$diag[1]\ddot{\mathbf{q}} + diag[2\zeta_k\Omega_k]\dot{\mathbf{q}} + diag\left[\Omega_k{}^2\right]\mathbf{q} + \sum_{j=1}^{6} g_j(t)\mathbf{r}_j\mathbf{r}_j^t\mathbf{q} = \sum_{j=1}^{6} k_j(t)\mathbf{r}_j\mathbf{r}_j^t\mathbf{q}_s \quad (3)$$

or

$$diag[1]\ddot{\mathbf{q}} + diag[2\zeta_k\Omega_k]\dot{\mathbf{q}} + diag\left[\Omega_k{}^2\right]\mathbf{q} + \sum_{j=1}^{6} g_j(t)\mathbf{r}_j\mathrm{E}_j(t) = \sum_{j=1}^{6} k_j(t)\mathbf{r}_j\mathrm{E}_{(s)j}(t) \quad (4)$$

In equations (3,4), $\mathbf{q} = \mathbf{Vx}$ is the modal co-ordinates vector, $\mathbf{r}_j = \mathbf{VR}_j$ is the modal structural vector at the j-th mesh, Ω_k is the k-th natural frequency, ζ_k is the corresponding modal viscous damping, $g_j(t) = k_j(t) - \overline{k_j}$ is the fluctuating part of meshing stiffnesses, $\mathrm{E}_j = \mathbf{r}_j^t\mathbf{q}$ is the dynamic transmission error and $\mathrm{E}_{(s)j} = \mathbf{r}_j^t\mathbf{q}_s$, the static ones at the j-th mesh.
To solve equation (4), we introduced an iterative process directly achieved in the spectral domain. After several judicious transformations, the iterative process is written as follows

$$E_i(\omega)^{(p+1)} = E_i(\omega)^{(0)} - \sum_{j=1}^{6} T_{ij}(\omega)[G_j(\omega)\otimes E_j(\omega)^{(p)}] \quad (5)$$

with

$$T_{ij}(\omega) = \sum_{k=1}^{N} \mathrm{r}_{ik}\mathrm{r}_{jk}H_k(\omega) \quad (6)$$

All the variables are expressed in the spectral domain by Fourier transform, the operator $\otimes$ denotes the convolution product, and $T_{ij}(\omega)$ is a function which only depends on the modal characteristics, in particular the classical frequency complex response functions $H_k(\omega)$.

4 RESULTS

After ensuring the method validity by comparisons with results obtained by Runge-Kutta time integration scheme, we have studied several test cases, including phase shifting effects between mesh stiffness fluctuations. As an example, we show in Figure 2 the evolution of the rms value of transmission errors versus the mesh frequency, obtained both by the Runge-Kutta method and our

Iterative Spectral method. We observe a very good agreement between methods. Further, one observes several dynamic amplifications that correspond to parametric resonances. These last can be easily interpreted by the knowledge of the spectral contents of responses and the mean modal characteristics of the geared system.

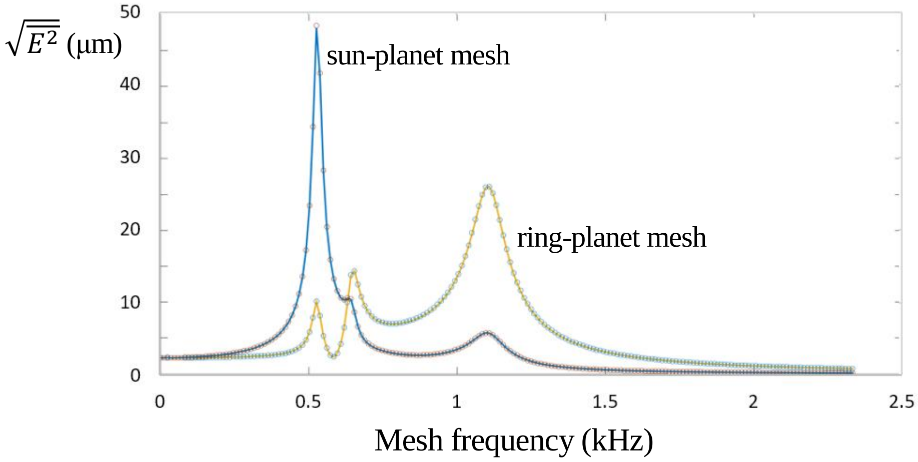

Figure 2. Rms values of Dynamic Transmission Errors versus the mesh frequency (continuous lines: Iterative Spectral Method; round marks: Runge Kutta Method).

Finally, results show that the main interest of the iterative spectral method is the very short computing time leading to very efficient simulation tools for predicting dynamic behaviour of planetary gear transmissions.

5 ACKNOWLEDGMENTS

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