A LAGRANGIAN BASED DAMAGE INDICATOR FOR USE ON COMPLEX STRUCTURES’

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ABSTRACT

Structural health monitoring has attracted much attention in many engineering fields. The four-level damage identification process: existence, localisation, severity and prediction of damage evolution, can be partly realised if a suitable indicator is chosen. In this paper a new ”Lagrangian” damage indicator for rib-reinforced plates is presented based on the structure’s frequency response function (FRF). This has been developed from the concepts of strain energy and curvature allowing for the creation of a Lagrangian indicator field. Damages and singularities present in the structure are characterised by a localised drop in value of the Lagrangian in the field. In the post-processing of results, the plate is split into multiple segments and the mean Lagrangian is calculated for each segment. This process has made the damage more apparent and easier to identify. These results show that the developed damage indicator is effective in detecting damages in rib-reinforced plate structures.

1 INTRODUCTION

Plate-like structures are widely used in various engineering applications, ranging from their use in the aerospace industry to the mechanical industry. As such, it is important to monitor the damage condition of plates to avoid unpredicted structural failure which may have severe consequences. In the field of structural health monitoring (SHM), several techniques have been proposed to monitor damage on various structures. Doebling [1] performed a thorough review of damage identification techniques, focusing on the changes of dynamic response of the structure. These are more commonly known as vibration-based methods. A method based on a structure's mode curvature shape was first demonstrated by Pandey [2] who successfully applied it to both cantilever and simply supported beams. Based on this concept, Wu [3] proposed an indicator based on the change in uniform load surface (ULS) curvature for two-dimensional plate structures using the mode shapes of the first few modes of a damaged and undamaged structures. Another damage indicator proposed by Navabian [4] was based on a plate's mode shape curvature as well as its displacement and slope. Cornwell [5] has also shown that the mode strain energy before and after being damaged can be calculated from its curvature. An indicator using spectral strain energy proposed by Bayissa [6] has been proven to be effective in detecting damage in plates. This damage indicator uses the curvature power spectral density and moment power spectral density of the plate. Sampaio [7] chose to use a plate's frequency response function (FRF) to calculate its curvature.

In this paper a new Lagrangian damage indicator is introduced, based on the concepts mention above. This damage indicator was tested through the use of a $0.6m \times 0.0005m \times 0.0085m$ Akyplac® plate numerically modelled in ANSYS. The plate was then separated into 20 segments to calculate the mean damage indicator for each segment.

2 DAMAGE INDICATORS

It has been previously proven that the curvature can be an effective indicator for damage identification in structures [2, 7], which is calculated by using the finite central difference approximation. When taking into account the kinetic, strain and elastic energies of the plate, the Lagrange damage indicator can be written as:

$$\frac{1}{2}mv^2 - \frac{1}{2}D[(\frac{\partial^2 u}{\partial x^2})^2 + (\frac{\partial^2 u}{\partial y^2})^2 + 2(1-\nu)(\frac{\partial^2 u}{\partial x \partial y})^2 + 2\nu\frac{\partial^2 u}{\partial x^2}\frac{\partial^2 u}{\partial y^2}] \tag{1}$$

However, this paper focuses only on the kinetic variables - curvature and velocity. The effect of mass and flexural stiffness were neglected as these variables are difficult to measure when a plate is in service. Calculations of the damage indicator at the specific point of damage as well as the damaged segment showed that they increased when the poisson's ratio was increased from 0 to 0.5. However, this increase did not affect the average segment values significantly. Thus, the Poisson's Ratio was set as 0 for all performed calculations, ignoring the elastic energy term. The damage indicator used in this paper is therefore defined as:

$$ind = velocity^2 - (\frac{\partial^2 u}{\partial x^2})^2 - (\frac{\partial^2 u}{\partial y^2})^2 - 2(\frac{\partial^2 u}{\partial x \partial y})^2 \tag{2}$$

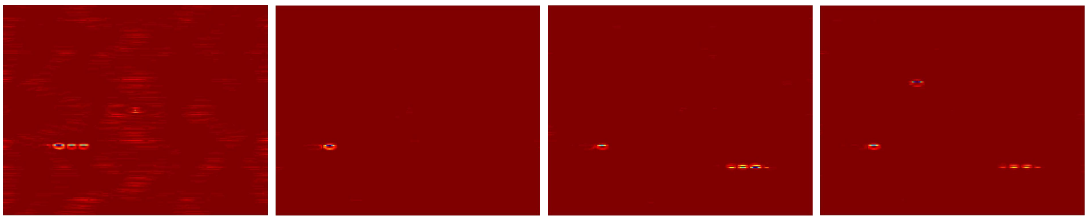
With the FRF of the structure, the distribution of the Lagrangian indicator field was obtained across the entire plate.

3 DAMAGE DETECTION ON NUMERICAL EXAMPLES

For the analyses, an Akyplac® twinwall polypropylene sheet was used. It is a lightweight durable corrugated plastic and is mostly used for packaging purposes. The model used for the dynamic analyses was a $0.6m \times 0.8m \times 0.01m$ plate, composed of a 0.75mm-thick upper and lower layer reinforced by 97 ribs with a thickness of 8.5mm. These ribs had the dimensions of $0.6m \times 0.0005m \times 0.0085m$ and were each separated by a 8.3mm gap. They were numbered sequentially from the bottom up and a distance of 1.6mm was between the first/last rib and its nearest edge. The SOLID185 elements in ANSYS were chosen for the modelling of both the layers and the ribs. Along the edges of plate, free boundary conditions were used and an excitation point was placed normally in the middle of plate.

Two types of damages were discussed: the damage in the ribs and the damage on the skin. It was found that the first Lagrangian-like indicator was more sensitive to the damage in the ribs. In the model used, there were 100mm-long damages in the 32th and 33th ribs with a distance of 100mm to the nearest edge. The presence and location of damage were able to be identified (Figure 1(a)) through a drops in the Lagrangian at the area of the damage. The lowest Lagrangian value was found to be -6.825 compared to the average plate Lagrangian value of -0.0587.

Numerical analyses on other damage cases were also performed. Figure 1(b) shows the indicator field with a 40mm-long damage in the 32th and 33th ribs where it can be seen that the zone with the reduced Lagrangian indicator has shortened. As such, this indicator is able to estimate the length of the damaged area, and hence, the severity of the damage. It should be noted that the Lagrangian value dropped to -49.171 at the damage zone compared to the plate average of -0.082. Another 100mm-long damage was created in the 23th and 24th ribs. The indicator field in Figure 1(c) shows the location and the length of damages. Furthermore, a study was performed with the two damages cases previously discussed and an additional 40mm-long damage in the 61th and 62th ribs (Figure 1(d)). From this, it has been shown that the indicator is able to identify multiple damaged areas on the same structure.



(a) Plate with a 100mm-long damage

(b) Plate with a 40mm-long damage

(c) Plate with a 40mm-long damage and a 100mm-long damage

(d) Plate with two 40mm-long damages and a 100mm-long damage

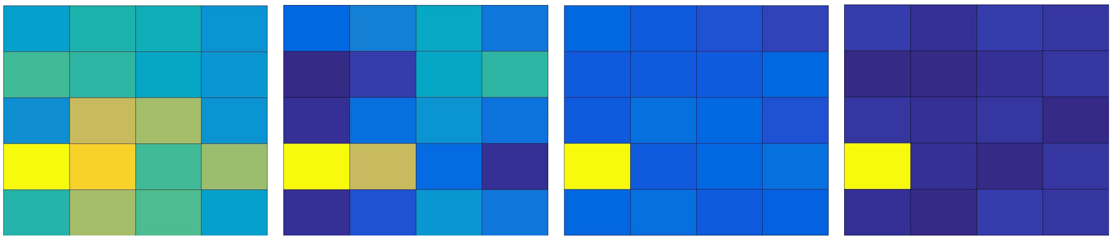
Figure 1. Lagrangian indicator field for different damage cases

4 RESULTS POST-PROCESSING

To make the results less susceptible to noise and other minor variations in the Lagrangian Indicator field, both the damaged and the undamaged plates were divided into 20 segments of $0.15m \times 0.16m$ each. The mean Lagrangian for each damaged segment was then calculated and the difference between it and its undamaged counterpart was found.

Two damage cases were used; the plate with a 100mm-long damage and the plate with a 40mm-long damage. Again, these damages have been characterised by a drop in the segment's

Lagrangian value. As can be seen in Figure 2, it is easier to identify the damage locations when it is compared to the original undamaged plate. The Lagrangian of the damaged segment also varied for the different damage cases. It was modelled to be at -0.4667 for the 40mm-long damage case and -0.1139 for the 100mm-long damage case.



(a) Plate with 100mm-long damage

(b) Plate with 100mm-long damage compared with undamaged plate

(c) Plate with 40mm-long damage

(d) Plate with 40mm-long damage compared with undamaged plate

Figure 2. Damaged plates split into segments and compared to undamaged plate

5 CONCLUSION

In this paper, a Lagrangian damage indicator based on the kinetic energy and curvature of the plate has been proposed. This damage indicator has been proven to be effective in detecting and locating the presence of one or multiple damage cases.

For future work on the subject, the relationship between the damage severity and the change in Lagrangian should be further investigated. This will allow for a more accurate measurement of the extent of damage.

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