AN EFFECTIVE FORMULATION AND A PHYSICAL DISCRETE MODEL FOR GEOMETRICALLY NONLINEAR TRANSVERSE VIBRATIONS OF A SYMMETRICALLY LAMINATED COMPOSITE BEAM

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ABSTRACT

The problem of geometrically nonlinear free vibration of symmetrically laminated composite clamped beams (SLCCB) is described by an N-dof discrete model of an equivalent isotropic beam, with effective bending and axial stiffness parameters. The model is made of (N +1) bars, connected by N masses placed at the bar ends, connected by rotational springs, presenting the beam flexural rigidity. The large transverse displacements of the bar ends induce a variation in their lengths giving rise to axial forces causing the nonlinear effect and modeled by longitudinal springs. The nonlinear vibration problem, defined in terms of the mass tensor m_{ij}, the linear rigidity tensor k_{ij} and the nonlinearity tensor b_{ijk},, is reduced, via application of Hamilton's principle, to a nonlinear algebraic system solved using an explicit method for calculating the (SLCCB) fundamental nonlinear mode and associated amplitude dependent frequency parameters. The numerical results are found to be in a good agreement with previously published results, based on a semi analytical composite beam continuous theory. The discrete system for the (SLCCB), developed and validated here, can be used in further applications to investigate nonlinear vibrations of non-uniform composite beams, with irregularities in the mass or in the stiffness distributions.

1 INTRODUCTION

In a series of previous works, it has been shown, both theoretically and experimentally, that beams constrained at both ends exhibit significant geometrical nonlinear behaviour at large vibration amplitudes, due to the axial strains induced by the large displacements. It has been shown also that composite structures exhibit a more accentuated nonlinear behaviour than those made of classical materials [1]. Symmetrically laminated clamped composite beams (SLCCB) are used in the design of many engineering structures such as aircrafts, space vehicles, and defence industries. Very often, they are subjected to high excitation levels in severe work environments inducing large vibration amplitudes. It is important in such situations, for obvious security and comfort reasons, that analytical and numerical tools are available, which enable designers to analyze and estimate accurately how far the structural dynamic characteristics deviate from those predicted by linear theory. In [2], the nonlinear homogeneous beam bending vibrations have been investigated using an N dof discrete system made of (N +1) bars, connected by N masses placed at the bar ends, connected by (N+2) rotational springs, presenting the beam flexural rigidity (see figure 1). The large transverse displacements of the ends of the bars, modelled by longitudinal springs (see figure 2), induce a variation in their lengths giving rise to axial forces causing geometrical nonlinearity. The analogy between the characteristics of the classical continuous beam model and those of the present discrete model was developed. The nonlinear vibration problem, defined in terms of the mass tensor m_{ij}, the linear rigidity tensor k_{ij} and the nonlinearity tensor b_{ijkl}, was reduced, via application of Hamilton's principle, to a nonlinear algebraic system solved using the so-called first formulation developed in [3]. The main advantage of nonlinear physical discrete models is their ability to be used quite easily to analyze the nonlinear behaviour of beams with irregularities in the geometry, mass or stiffness distributions. It was then interesting to examine the extension of the discrete model to inhomogeneous beams, such as the (SLCCB) examined in the present work. The approach adopted is based on the combination of a homogenization procedure [4, 5] with the N dof discrete model [2] to obtain an equivalent homogeneous beam with effective bending and axial stiffness parameters.

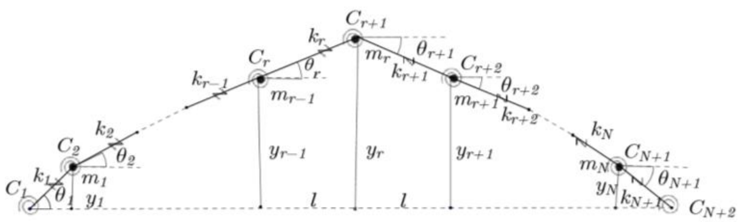


Figure1: The N dof discrete model of the (SLCCB)

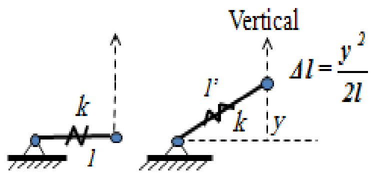
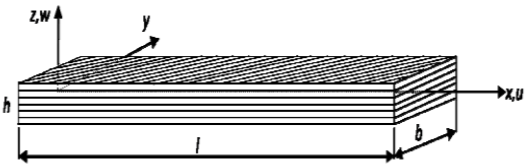


Figure 2: Nonlinear effect due to the Pythagorean Theorem

2 DISCRETE FORMULATION AND NUMERICAL RESULTS

The (SLCCB) studied in [4], [5] and [6] and examined in this work (Figure 3) has the following geometrical and mechanical characteristics: h = 0,001; b = 0,01; L = 0,25m; E_1=155 GPa, E_2=21,1GPa, ρ=1560 Kg/m3, υ_{12} = 0,248). The intermediate parameters and Lay-up allowing calculation of the equivalent isotropic beam parameters, i.e. $(ES)_{eff} = bA_{11}$ and

$(EI)_{eff} = b(D_{11} + (B_{11}^2/A_{11}))$, which are the effective axial and bending stiffness respectively, for the four composite beams considered in the present paper are given in Figure 3.



	A_{11}	B_{11}	D_{11}	$(ES)_{eff}$	$(EI)_{eff}$
Unidirectional	155749200	0	12.979	1557492	0.1298
(0/90/90)s	59997696	0	9.4336	599977	0.0943
(90/90/0)s	59945424	0	1.4564	599454	0.0145
Cross-ply	12120000	0	1.0100	121200	0.0101

Figure 3: Laminated beam notation and characteristics

Consider the N dof discrete system developed in [1] for an isotropic beam. The nonlinear differential equations governing the system nonlinear dynamics is written in the displacement basis (DB) in a matrix form as follows:

$$[K]\{A\} - \omega^2 [M]\{A\} + [B(A)]\{A\} = \{0\} \quad (1)$$

A discretization procedure, similar to that developed in [1], is applied to the equivalent isotropic beam using the parameters $(ES)_{eff}$ and $(EI)_{eff}$ calculated as functions of the composite beam stiffness coefficients A_{11}, B_{11} and D_{11} by: $(ES)_{eff} = bA_{11}; (EI)_{eff} = b(D_{11} + (B_{11}^2/A_{11}))$ [4, 5].The effective parameters are inserted in the rigidity matrix $[K]$ and the nonlinear rigidity tensor $[B(A)]$ presenting the discrete system through the rotational and longitudinal spring stiffness defined by: $C = \frac{(EI)_{eff}}{l}$ and $k = \frac{(ES)_{eff}}{l}$. It should be noted that the calculations, based on the so-called first formulation presented in [3], are performed in the modal basis (MB), in order to yield good estimates of the (SLCCB) amplitude dependent nonlinear frequencies using the single mode approach (SMA), giving: $(\omega_{\text{disc}}^{\text{nl}})^2 = \frac{\overline{k_{11}}}{\underline{m_{11}}} + \frac{3}{2}\frac{\overline{b_{1111}}}{\underline{m_{11}}} a_1^2$. For validation purposes, the numerical results, based on equation (1), for (SLCCB) vibration amplitudes up to 1.5 times the beam thickness are presented in Figure 4 for four composite beams and compared to those of references [5], [6] showing a satisfactory agreement. For higher amplitudes, the so-called second formulation developed in [3], which is known to have a wider range of validity, may be used as an alternative to the iterative method for solving the nonlinear amplitude equation.

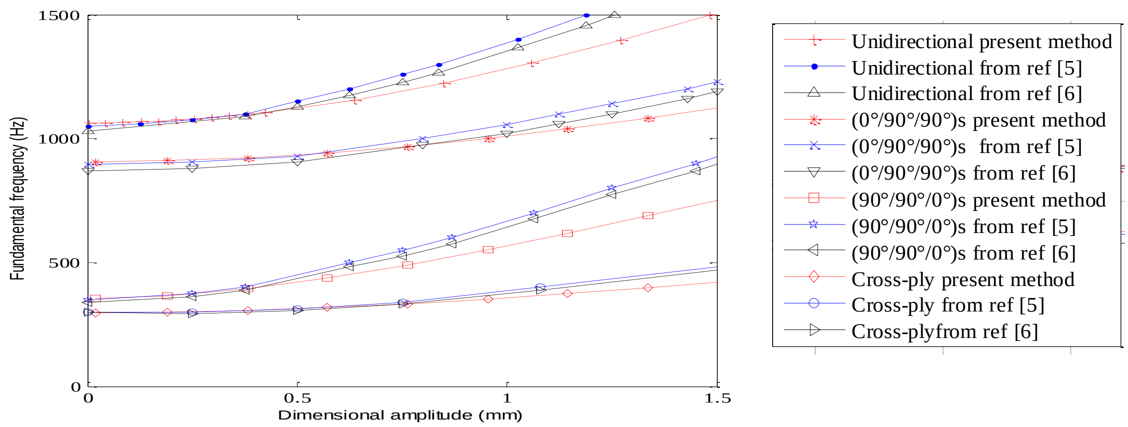


Figure 4: Comparison of the nonlinear frequencies obtained for different (SLCCB) lay-up by the present discrete model with previously published results

3 Conclusion

The problem of geometrically nonlinear vibrations of (SLCCB) is described by an *N*-dof discrete model of an equivalent isotropic beam, with effective bending and axial stiffness parameters. The model is made of (N +1) bars, connected by N masses placed at the bar ends, connected by rotational springs, presenting the beam flexural rigidity. The large transverse displacements of the bar ends induce a variation in their lengths giving rise to axial forces causing the nonlinear effect and modelled by longitudinal springs. The nonlinear vibration problem, defined in terms of the mass tensor m_{ij}, the linear rigidity tensor k_{ij} and the nonlinearity tensor b_{ijkl}, is reduced, via application of Hamilton's principle, to a nonlinear algebraic system solved using an explicit method for calculating the (SLCCB) fundamental nonlinear mode and associated amplitude dependent frequencies. The numerical results are found to be in a good agreement with previously published results, based on a semi analytical composite beam continuous theory. As has been done with isotropic beams in [4, 5 and 6], the discrete system for the (SLCCB), developed and validated here, may be used in further applications to investigate nonlinear vibrations of non-uniform composite beams, carrying point masses, or beams with irregularities in the mass [7] or in the stiffness distributions.

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