



## DESIGN AND EXPERIMENTAL VALIDATION OF A HIERARCHICAL AUXETIC RECTANGULAR PERFORATED METAMATERIAL

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### ABSTRACT

*The understanding of wave propagation in a metamaterial with hierarchical, auxetic rectangular perforations is presented in this work. The metamaterial is a 2D structure with chain-ing horizontal and vertical perforations exhibiting auxetic in-plane behaviour. Some numerical eigenvalue tools are used for the dispersion analysis of this structure. It is first observed that the total width of Band gaps increases with the hierarchy. In order to validate the design of the metamaterial, results issued from a full 3D model of a finite structure embedding an interface composed by a distributed set of the unit cells are presented. After this step, a comparison between the results obtained using the structure simulation and the experimental results are presented with critical analysis.*

## 1 INTRODUCTION

A periodic medium is a material or a structural system that exhibits spatial periodicity. The study of periodic structures has a long history in the field of vibrations and acoustics [1]. This topic has interested researchers over the years, and a growing activity on this field is observed on the last years, with the objective of designing structures exhibiting properties that conventional ones cannot possess [2]. Dynamical behaviour analysis of plates with hierarchical, auxetic rectangular perforations are described in this paper.

## 2 GEOMETRY OF THE HIERARCHICAL PERFORATED AUXETIC LATTICE

Figure 1a shows the structural dimensions of the square lattice with rectangular perforations. The symmetry of the geometry in the  $x - y$  plane allow to define the entire geometry of the unit cell using only 2 parameters: the void aspect ratio,  $AR = a/b$  and the intercell spacing  $S$  [3].

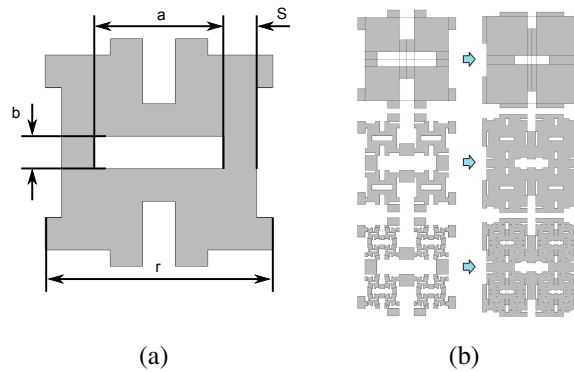


Figure 1: a) Geometry parameters of the base unit cell. b) Hierarchical, auxetic rectangular perforations at Level 1, 2 and 3 with  $AR = 4$  and from  $S = 0.2$  to  $S = 0.8$ .

As a reference, the level 1 [3] is compared with the hierarchical levels 2 and 3. At level 1, 4 rigid squares are present in the unitcell. In each square, the reference structure is used by applying a scale ratio to obtain the level 2. Exactly the same at level 3, in this subunit. The parametric analysis is carried out with the aspect ratios ( $AR$ ), the intercell spacing ( $S$ ) and the level of hierarchy. Figure 1b shows how intercell spacing change in both levels of hierarchy. Voids are larger than a low parameter  $S$  and the porosity increases with the level of hierarchy.

## 3 DYNAMIC PROPERTIES

### 3.1 Dispersion analysis

Properties are calculated using the finite element model of the unit cell with Floquet-Bloch periodic conditions applied on the borders of the domain [4].

The results of the analysis correspond to dispersion diagrams which only provides information on the contour of the Brillouin zone allowing identification of the bandgaps. Hence, only specific directions are investigated.

Band gaps are observed at some specific values of  $AR$  and  $S$  (see figure 3).

These band gaps are called omnidirectional band gap because whatever the direction of the wave propagation, this wave can not propagate. In our case, a particular interest is given to omnidirectional band gaps. To compare the results of the eigenvalue analysis at different levels we have computed for each geometry configuration of the perforated composite plate

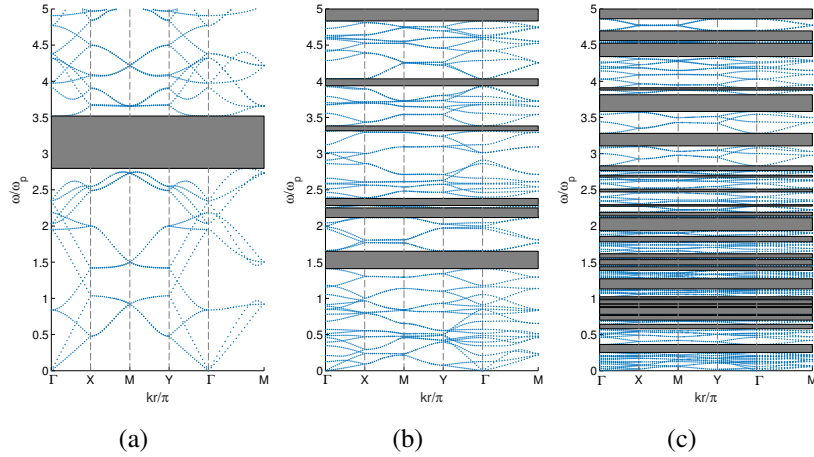


Figure 2: Dispersions in the  $k$  space for the lattice with  $AR = 4$ ,  $S = 0.3$  for a) Level 1, b) Level 2 and c) Level 3.

the equivalent volume fraction ( $\phi$ ) and computed by Finite Element the natural frequency of a rectangular plate (plane stress) with the same overall dimension of the lattice with Poisson's ratio equal to the one of the core material ( $\nu_c$ ), scaled density  $\bar{\rho} = \phi\rho_c$  and equivalent Young's modulus  $\bar{E} = E_c\phi^2$ . The resulting fundamental frequency is denominated as  $\omega_p$ . The modal density increase with the hierarchy, it is true whatever the value of the parameter  $S$  is. The reader is invited to refer to a previous article [5] for details.

### 3.2 Finite structure

The main goal of this section is to validate in a finite structure the phenomenon observed on an infinite structure. A finite element model is presented. This is followed by an experimental validation. The metamaterial ( $68 \times 28 \times 0.4 \text{ cm}^3$ ) includes an interface composed by  $4 \times 4$  unit cells ( $7 \times 7 \times 0.4 \text{ cm}^3$ ) (figure 3a). The metamaterial is made in acrylic which properties are  $E = 3.01 \text{ GPa}$ ,  $\nu = 0.375$  and  $\rho = 1190.25 \text{ kg/m}^3$  with a loss factor ( $\eta$ ) equal to 4.2%.

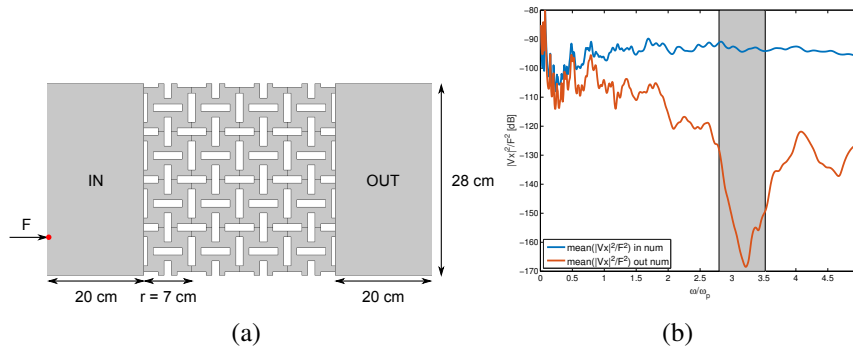


Figure 3: a) Finite structure with an interface composed by 16 unit cells for level 1. The point load is marked by a red dot. b) Numerical frequency responses for level 1. Average squared velocity amplitude  $|Vx|^2$  for the input plate (IN) and the output plate (OUT) respectively in blue and red. The grey shape represent the bandgap predicted by the dispersion analysis.

Numerical frequency responses for level 1 are presented in figure 3b, squared velocity amplitudes  $|Vx|^2$  are averaged for the input plate (IN) and the output plate (OUT). The bandgap

predicted by the dispersion analyse is represented by a grey shape. An output attenuation is well observed in the frequency range predicted by bandgap.

#### 4 EXPERIMENTAL VALIDATION

The metamaterial is realised by laser cutting in a whole 4 mm acrylic glass plate. This experimental validation is a real challenge. Boundary conditions need to respect the plane stress condition. Figure 4 illustrates the experimental facility with the metamateriel, its bracket, the vibrometer, the force sensor and the accelerometer. A shaker provides clean harmonic excitation up to 5 kHz.

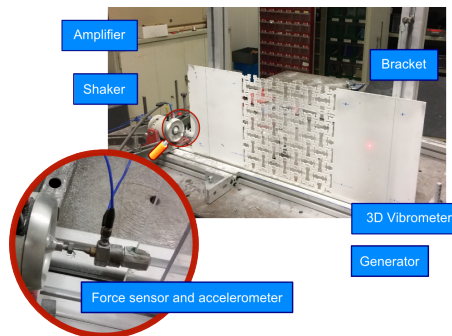


Figure 4: Experimental facility with the main equipment as the metamateriel and the vibrometer. A zoom is done on the excitation system, a shaker instrumented with force sensor cell and accelerometer.

#### 5 CONCLUSION

This study shows the possibility of creating bandgaps by simple cutting in plane structures. The damping is important, an overall smoothing of the frequency response functions is observed and a very large attenuation between the input and the output of the network. The experimental setup was a real challenge. The results engage a critical analysis which highlight the strengths and weaknesses of this experiment.

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