WAVE FINITE ELEMENTS - FINITE ELEMENTS COUPLING TO COMPUTE THE DYNAMIC RESPONSE OF AN HETEROHENEOUS RAILWAY TRACK

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ABSTRACT

Railway noise is a critical issue concerning environmental noise. At the wheel/rail contact point, both the wheel and the track are dynamically excited and vibrate together to emit the well-known rolling noise within a frequency range comprised between 100 Hz to 5000 Hz. The point receptance of the rail is an important quantity to accurately predict wheel-rail noise emission. The goal of this paper is to compute the dynamic behaviour of an heterogeneous railwaytrack using a biperiodicity method on a heterogeneous unit cell. A coupling between the Finite Element Method (FEM) and the Wave Finite Element Method (WFEM) is made to model all kinf of heterogeneities along the track. An example is applied by modelling the heterogenities as elastic supports, an external force is applied inside the unit cell and the response gives good agreements with experimental results from litterature.

1 INTRODUCTION

Railway noise is a critical issue concerning environmental noise. At the wheel/rail contact point, both the wheel and the track are dynamically excited and vibrate together to emit the well-known rolling noise within a frequency range comprised between 100 Hz to 5000 Hz. The track is made of a rail supported by an elastomeric pad, a sleeper and a damp resilient ballast layer. The point receptance of the rail is an important quantity to accurately predict wheel-rail noise emission. The theory of periodicity developed by Mead [1] has been widely used to compute the response of heterogeneous infinite railwaytracks [2] i.e which are periodically supported. His theory allowed the use of finite elements [1, 3] to describe the dynamic behaviour of infinite periodic structures with arbitrary-shaped sections.

In this paper, the dynamic behaviour of an heterogeneous railwaytrack is computed using a biperiodicity method [1], through a coupling between the Finite Element Method (FEM) and the Wave Finite Element Method (WFEM) [4]. This coupling allows the use of all kinds of heterogeneities along the track. The principle of the WFEM is first recalled, then the forced response is computed inside a unit cell by using a biperiodicity method.

2 MODELLING APPROACH

2.1 The Wave Finite Element Method for periodic structures

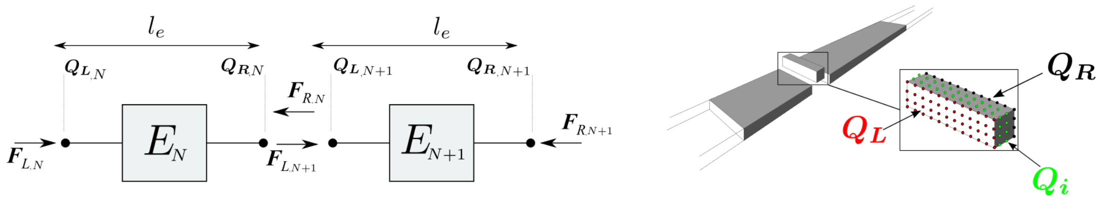


FIGURE 1. Displacements and forces applied on a unit cell of length l_e

The periodic structure is an infinite series of unit cells coupled with n_c degrees of freedom. The coupling coordinates are $\boldsymbol{Q}_L$ $\boldsymbol{Q}_R$ and the coupling forces $\boldsymbol{F}_L$ $\boldsymbol{F}_R$. The dynamic stiffness matrix $\boldsymbol{D}$, assembled with the finite element method, can be condensed [3] by expressing through a matrix inversion the internal coordinates $\boldsymbol{Q}_i$ in function of the coupling coordinates such as :

$$\begin{bmatrix} \boldsymbol{D}_{LL} & \boldsymbol{D}_{LR} \\ \boldsymbol{D}_{RL} & \boldsymbol{D}_{RR} \end{bmatrix} \begin{Bmatrix} \boldsymbol{Q}_L \\ \boldsymbol{Q}_R \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F}_L \\ \boldsymbol{F}_R \end{Bmatrix} \tag{1}$$

The use of the periodicity principle [3**?**] leads to a generalised eigenvalue problem which gives a set of $2n_c$ couples $(\boldsymbol{\Theta}_j, \Lambda_j)$ for each frequency. $\boldsymbol{\Theta}$ represents the waveshape basis splitted into waveshapes displacements $\boldsymbol{\Theta_q}$ and waveshapes forces $\boldsymbol{\Theta_f}$. The eigenvalue Λ is associated with the propagation constant γ and the periodic lenght l_e of the unit cell such as [1, 3] :

$$\Lambda_j = e^{\gamma_j \, l_e} \qquad \boldsymbol{\Theta}_j = [\boldsymbol{\Theta}_{\boldsymbol{q}j} \;\; \boldsymbol{\Theta}_{\boldsymbol{f}j}]^T \tag{2}$$

The basis can be splitted into positive propagative direction if $|\Lambda_j| < 1$ associated with n_c $\boldsymbol{\Theta}^+ = [\boldsymbol{\Theta}^+_{\boldsymbol{q}} \;\; \boldsymbol{\Theta}^+_{\boldsymbol{f}}]^T$ and into negative propagative direction if $|\Lambda_j| > 1$ with n_c $\boldsymbol{\Theta}^- = [\boldsymbol{\Theta}^-_{\boldsymbol{q}} \;\; \boldsymbol{\Theta}^-_{\boldsymbol{f}}]^T$. The forced response of a periodic structure can be written as a sum of these waves [3].

2.2 Computation of the forced response using a biperiodicity method

The periodicity theorem is applied on a $0.6m$ length cell (named *Unit Cell I*) which represents the distance between two elastic supports of a railway track. As reminded in the previous section, this cell needs to be condensed at its coupling interfaces. A biperiodicity method [1] is used to condensate the homogeneous part (made of several slices of the rail named *Unit Slice*), it consists in expressing the condensed dynamic stiffness matrix of this part thanks to the wave-shape basis [3]. The heterogenous parts (which can contain the elastic supports) are modelled with two FEM parts and are coupled with the homogeneous part [4]. Unit cells I are associated

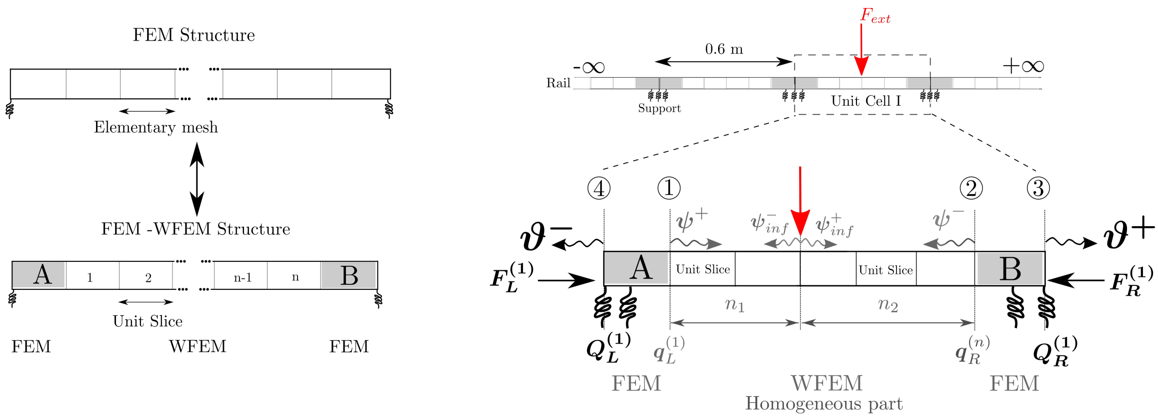


FIGURE 2. Waves inside and outside a discretized unit cell I made of a FEM-WFEM coupling

with the waveshapes $\boldsymbol{\Theta}$ of amplitudes $\boldsymbol{\vartheta}$, unit slices are associated with the waveshapes $\boldsymbol{\Phi}$ of amplitudes $\boldsymbol{\psi}$ and with the eigenvalue $\boldsymbol{\lambda}$ (such as $|\lambda| > 1$). The displacements and the forces inside the internal waveguide can be written as a sum of infinite waves $\boldsymbol{\psi}_{inf}$ i.e waves propagating in an infinite structure and of reflected waves $\boldsymbol{\psi}$ on the boundaries. The modal waveshape amplitude $\boldsymbol{\psi}_{inf}$ can be calculated as [1] :

$$\begin{Bmatrix} \boldsymbol{\psi}^+_{inf} \\ \boldsymbol{\psi}^-_{inf} \end{Bmatrix} = \begin{bmatrix} -\boldsymbol{\Phi_f}^- & -\boldsymbol{\Phi_f}^+ \\ \boldsymbol{\Phi_q}^- & \boldsymbol{\Phi_q}^+ \end{bmatrix}^T \begin{Bmatrix} 0 \\ F_{ext} \end{Bmatrix} \tag{3}$$

The numbers of unit slices on the left n_1 and on the right n_2 of the external force are such as $n_1 + n_2 = n$. The displacements $\boldsymbol{q}_L^{(k)}$ and the forces $\boldsymbol{f}_L^{(k)}$ of the unit slice k inside the homogeneous part and $\boldsymbol{Q}_L^{(1)}$ $\boldsymbol{F}_L^{(1)}$ those of the unit cell I interfaces can be written as :

$$\boldsymbol{q}_L^{(k)} = \boldsymbol{\phi}_{\boldsymbol{q}}^+ \boldsymbol{\lambda}^{k-1} \boldsymbol{\psi}^+ + \boldsymbol{\phi}_{\boldsymbol{q}}^- \boldsymbol{\lambda}^{n+1-k} \boldsymbol{\psi}^- + \boldsymbol{\phi_q}^- \boldsymbol{\lambda}^{n_1+1-k} \boldsymbol{\psi}^-_{inf} \tag{4}$$

$$\boldsymbol{f}_L^{(k)} = \boldsymbol{\phi}_{\boldsymbol{f}}^+ \boldsymbol{\lambda}^{k-1} \boldsymbol{\psi}^+ + \boldsymbol{\phi}_{\boldsymbol{f}}^- \boldsymbol{\lambda}^{n+1-k} \boldsymbol{\psi}^- + \boldsymbol{\phi_f}^- \boldsymbol{\lambda}^{n_1+1-k} \boldsymbol{\psi}^-_{inf} \tag{5}$$

$$\boldsymbol{Q}_L^{(1)} = \boldsymbol{\Theta}_{\boldsymbol{q}}^- \boldsymbol{\vartheta}^- \quad \boldsymbol{F}_L^{(1)} = \boldsymbol{\Theta}_{\boldsymbol{f}}^- \boldsymbol{\vartheta}^- \quad \boldsymbol{Q}_R^{(1)} = \boldsymbol{\Theta}_{\boldsymbol{q}}^+ \boldsymbol{\vartheta}^+ \quad \boldsymbol{F}_R^{(1)} = \boldsymbol{\Theta}_{\boldsymbol{f}}^+ \boldsymbol{\vartheta}^+ \tag{6}$$

The dynamic stiffness matrices of the A and B FEM parts are noted $\boldsymbol{D}_A$ $\boldsymbol{D}_B$, the dynamic equilibrium on the four interfaces (see Figure 2) gives after some rearrangements :

$$\begin{Bmatrix} \boldsymbol{\psi}^+ \\ \boldsymbol{\psi}^- \\ \boldsymbol{\vartheta}^+ \\ \boldsymbol{\vartheta}^- \end{Bmatrix} = \begin{bmatrix} \boldsymbol{I} & -\boldsymbol{R}^+\boldsymbol{\lambda}^n & 0 & \boldsymbol{X}^{-1}\boldsymbol{D}_{A,RL}\boldsymbol{\Theta}_{\boldsymbol{q}}^- \\ -\boldsymbol{R}^-\boldsymbol{\lambda}^n & \boldsymbol{I} & \boldsymbol{Y}^{-1}\boldsymbol{D}_{B,LR}\boldsymbol{\Theta}_{\boldsymbol{q}}^+ & 0 \\ \boldsymbol{Z}^{-1}\boldsymbol{D}_{B,RL}\boldsymbol{\lambda}^n & \boldsymbol{Z}^{-1}\boldsymbol{D}_{B,RL} & \boldsymbol{I} & 0 \\ \boldsymbol{W}^{-1}\boldsymbol{D}_{A,LR} & \boldsymbol{W}^{-1}\boldsymbol{D}_{A,LR}\boldsymbol{\lambda}^n & 0 & \boldsymbol{I} \end{bmatrix}^{-1} \begin{Bmatrix} \boldsymbol{R}^+\boldsymbol{\lambda}^{n_1}\boldsymbol{\psi}^-_{inf} \\ \boldsymbol{R}^-\boldsymbol{\lambda}^{n_2}\boldsymbol{\psi}^+_{inf} \\ -\boldsymbol{Z}^{-1}\boldsymbol{D}_{B,RL}\boldsymbol{\lambda}^{n_2}\boldsymbol{\psi}^+_{inf} \\ -\boldsymbol{W}^{-1}\boldsymbol{D}_{A,LR}\boldsymbol{\lambda}^{n_1}\boldsymbol{\psi}^-_{inf} \end{Bmatrix} \tag{7}$$

$$\boldsymbol{W} = [\boldsymbol{D}_{A,LL}\,\boldsymbol{\Theta}_{\boldsymbol{q}}^- - \boldsymbol{\Theta}_{\boldsymbol{f}}^-] \qquad \boldsymbol{X} = [\boldsymbol{D}_{A,RR}\,\boldsymbol{\phi_q}^+ + \boldsymbol{\phi_f}^+] \tag{8}$$

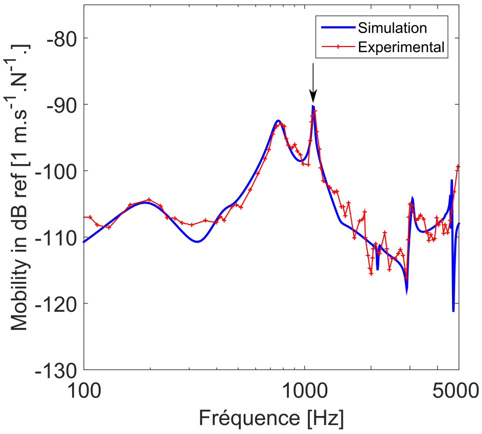
$$\boldsymbol{Y} = [\boldsymbol{D}_{B,LL}\,\boldsymbol{\phi_q}^- - \boldsymbol{\phi_f}^-] \qquad \boldsymbol{Z} = [\boldsymbol{D}_{B,RR}\,\boldsymbol{\Theta}_{\boldsymbol{q}}^+ + \boldsymbol{\Theta}_{\boldsymbol{f}}^+]$$

$$\boldsymbol{R}^- = [\boldsymbol{D}_{B,LL}\,\boldsymbol{\phi_q}^- - \boldsymbol{\phi_f}^-]^{-1}[\boldsymbol{D}_{B,LL}\,\boldsymbol{\phi_q}^+ - \boldsymbol{\phi_f}^+] \qquad \boldsymbol{R}^+ = [\boldsymbol{D}_{A,RR}\,\boldsymbol{\phi_q}^+ + \boldsymbol{\phi_f}^+]^{-1}[\boldsymbol{D}_{A,RR}\,\boldsymbol{\phi_q}^- + \boldsymbol{\phi_f}^-] \tag{9}$$

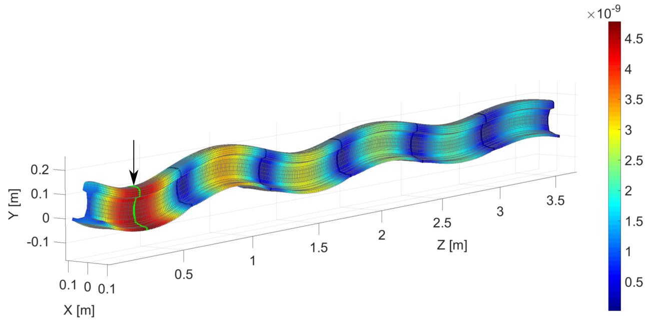
The first two lines of Equation (7) are the coupling between the heterogeneous parts and the homogenous one, the last two lines are the coupling between the heterogeneous parts and the semi-infinite structures.

3 RESULTS

The method is applied on an heterogenous railway track periodically supported. The rail section is meshed with linear hexahedral elements, the 0.6m length cell is divided into 60 slices i.e a 1cm elementary mesh. Elastic support parameters are taken as given by L.Gry [5]. The Figure 3(a) represents the mobility of the track due to an external vertical force applied at the middle of the unit cell I. The simulation gives a good correspondance with experimental results from [5] and well reproduces the periodicity effects. The Figure 3(b) represents the receptance along an half-track at the pinned-pinned frequency (1090Hz) which appears when half the wavelength is equal to the distance between supports.



(a)



(b)

4 CONCLUSION

In this paper, a method to compute the response of a heterogeneous periodic structure due to an external force applied in a unit cell has been proposed. It uses a FEM-WFEM coupling to model all kinds of heterogeneities along the waveguide. The method has been applied on an heterogeneous railway track laid on elastic supports and gives good agreements with experimental results.

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