ANALYSIS OF LARGE AMPLITUDE OSCILLATIONS OF SIMPLY SUPPORTED BEAMS THROUGH THE USE OF THE NONLINEAR NORMAL MODES (NNM) METHOD

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ABSTRACT

Simply supported beams are usually classified into two groups, depending on whether longitudinal displacements are restrained or not. This work goes deeper into the fact that the nonlinear behavior of the beam is significantly different in these two cases: one in which axial motion is allowed at one end but restricted at the other; and another in which there is no restriction to axial displacements at both ends. The analytical treatment of the problem leads to a relation between nonlinear frequency and amplitude for the different modes of vibration of the beam. A well-known commercial finite element software is used to validate the results of the analytical models. Nonlinear normal mode (NNM) shapes may be represented as a combination of several linear ones. The results of this investigation show that the contribution of linear modes other than the first one to each nonlinear one is significant. Different simulations are conducted with the aim to provide recommendations for the need of including such modes.

1 INTRODUCTION

The dynamic behaviour of linear elastic beams under hypothesis of small strains and small displacements is well-known. However, in numerous applications, deflections are large enough to make the assumption of small displacements no more suitable. In these cases, the equilibrium needs to be imposed on the deformed configuration of the structure, what makes the system nonlinear.

Although nonlinear vibrations of beams have been widely studied, available results are sometimes unclear and often contradictory [1[5]. For this reason, the present article intends to get some insight into the physical phenomenon and quantify the effect of the mentioned nonlinearities on the dynamics of simply supported beams with moderately large displacements.

Simply supported beams are usually classified into two groups, depending on whether longitudinal displacements are restrained or not. This article deals with both groups separately. It will be shown that the nonlinear behaviour of the beam is strongly different in these two cases: one in which axial motion is allowed at one end (unsymmetrical case) and restricted at the other and another one where there is no restriction to axial displacements (symmetrical case).

The analytical treatment of the simply supported axially unrestrained beam problem leads to a relation between nonlinear frequency and amplitude for the different modes of vibration of the beam. Some finite element simulations are carried out in order to validate the results, which are also compared to those obtained by other authors.

Later, the axially restrained case, where both ends of the beam are immovable, is also briefly studied. The aim of this part is to cast light on the question about whether nonlinearities other than midline stretching should or not be included in the model. Once again, different results can be found in the literature in this regard [1[5, [6, [7].

The analytical treatment in this work uses the concept of Nonlinear Normal Modes (NNMs) introduced by Rosenberg in the 60s [8], which has experienced a great development since 1990 due to the works of Pierre, Shaw, Vakakis, etc. [9, [10]. In short, for an unforced conservative system, a NNM can be defined as a family of periodic motions which occur onto a 2D invariant manifold in the phase space of the system. This manifold passes through a stable equilibrium point and, at that point, is tangent to one of the Linear Normal Modes (LNMs) of the linearized system. Then, NNMs are a natural generalization of LNMs, suitable to Nonlinear Systems. For a detailed exposition on NNMs, the reader is referred to [11].

2 SIMPLY SUPPORTED BEAM WITH UNRESTRAINED AXIAL DISPLACEMENTS

The procedure followed by Nayfeh in [6 [12] for obtaining the NNMs of continuous systems has been used in this investigation. **Fig. 1** show Frequency-Amplitude curves for the first NNMs and the configurations of axially unrestrained simply supported beams. Obviously, we are not taking into account the rigid body mode present in the symmetrical case.

In both figures, the blue curve corresponds to an analytical model, while the blue circles correspond to Finite Element results. We have used commercial program Abaqus®, discretizing the beam in 16 elements with cubic interpolation. The initial conditions for these Finite Element simulations have been chosen to correspond to one particular NNM. For the first NNM, we have also included some results from the literature.

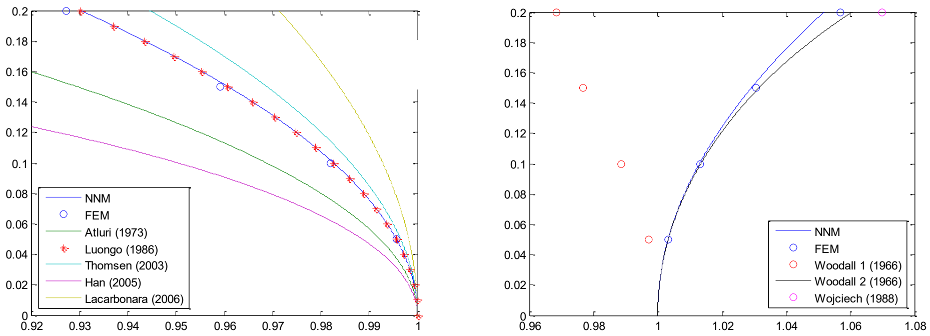

Fig. 1 frequency vs amplitude curves for the first NNM: unsymmetrical case (left) symmetrical case (right)

Fig. 2 shows in blue the deformed shapes for the first NNMs, including the contributions of the first 5 linear modes. Rigid body motion in the symmetrical case has been avoided.

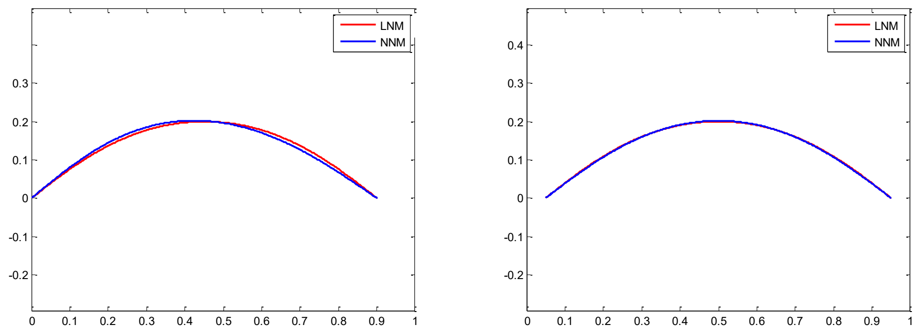

Fig. 2 Deformed shape corresponding to the first NNM: Unsymmetrical case (left) and Symmetrical case (right)

The main issue about a nonlinear frequency-amplitude curve (usually called *Backbone Curve*) is whether it shows hardening or softening behavior. It can be observed that, for the first mode, the unsymmetrical beam softens, while the symmetrical one hardens. The first immediate consequence is that, when dealing with a simply supported beam, it is not enough to specify whether axial motion is restrained or not since, even within the axially unrestrained group there exist different kinds of behavior.

3 CONCLUSIONS

It is found that, in the axially unrestrained case, two kinds on nonlinearities influence the motion of the beam. One is of geometric nature, while the other is due to longitudinal inertia. For the simply supported beam they produce, respectively, hardening and softening, but this may be different for other boundary conditions.

Two different configurations of axially unrestrained simply supported beams have been considered, one having a fixed end and other with both ends free in the longitudinal direction. They have been shown to exhibit different behaviours, suggesting that the usual distinction between axially restrained and unrestrained simply supported beams [5] is not enough for characterizing their dynamics. For the first NNM, the beam undergoes hardening in the symmetrical case and softening in the unsymmetrical case.

The reasonably good accordance between analytical and Finite Element results (with axially extensible elements) indicates that the assumption of inextensible middle line, used for the axially unrestrained case, is pertinent –at least for the first two NNMs–.

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