GEOMETRICALLY NON-LINEAR VIBRATIONS OF BEAMS CARRYING A POINT MASS AND RESTRAINED BY TRANSLATIONAL AND ROTATIONAL SPRINGS AT THE ENDS.

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ABSTRACT

The non-linear vibrations of a beam carrying a point mass at an arbitrary location and restrained by translational and rotational springs at the two ends are investigated analytically and a parametric study is performed, allowing examination of all possible combinations of classical restrained end conditions, including elastic restraints. The dynamic equation was written at two intervals of the beam span with appropriate end and continuity conditions. After the necessary algebraic transformations, the generalised transcendental frequency equation was solved iteratively using the Newton Raphson method. Once the corresponding program implemented, investigations have been made of the changes in the beam frequencies and mode shapes for many values of the mass, mass location and spring stiffness. Numerical results and plots have been given here of the beam frequencies and first mode shape corresponding to various situations. The effect of geometrical non-linearity has then been investigated using a semi analytical method based on Hamilton's principle and spectral analysis leading to solution of a non-linear amplitude equation. A single mode approach, performed in the modal basis, has been adopted in order to obtain, for various configurations of the beam examined, the backbone curves giving the amplitude dependent nonlinear frequencies.

1   INTRODUCTION

The operation of machines (machine tools, automotive, robot manipulators and others) introduces dynamic constraints on the various components of the engine and the supporting elements. To ensure correct operation, reduce the induced noise, and increase the machine fatigue life, it is essential to determine the natural frequencies of the system, in both the linear and non-linear regimes. Many such situations may be modeled by a beam carrying one or many masses, restrained at its ends by flexible rotational and translational springs [1-2]. This makes it possible to study different types of restrained end conditions, such as simply supported or clamped, depending on the values assigned to the spring stiffness. The vibrations of restrained beams supporting point masses have been partially examined before [1-3] but all of the studies available are restricted to the linear case. The present paper is based on a systematic parametric study allowing easy choice of the position of the mass to be added in order to adapt the linear frequencies and avoid possible resonances. In the second part, the effect of geometrical nonlinearity on the system "beam + added mass" amplitude dependent nonlinear frequencies is investigated. A single mode approach is adopted, combining the semi analytical method for nonlinear structural vibrations developed previously [4] and the linear modes calculated in the first part, and allowing various backbone curves to be drawn, corresponding to various values of the spring stiffness and added mass.
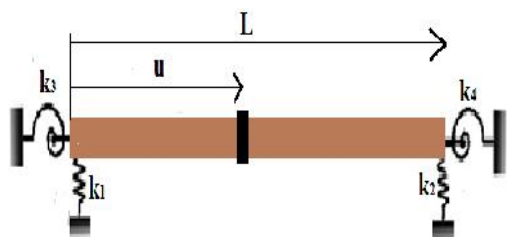

Figure. 1. The restrained beam with a point mass. Beam characteristics (L, S, ρ, E, I)

	$K_3=10\ k_4=10$		$K_3=100\ k_4=100$		$K_3=10\ k_4=100$	
η	M=0,5	M=1	M=0,5	M=1	M=0,5	M=1
0	4,15566	4,15566	4,64132	4,64132	4,39001	4,39001
0,1	4,10204	4,04572	4,61416	4,58502	4,31999	4,24643
0,2	3,89071	3,66687	4,40167	4,17988	4,06073	3,79719
0,3	3,64396	3,32802	4,07685	3,72112	3,78953	3,44078
0,4	3,48471	3,13988	3,85720	3,46419	3,63523	3,26359
0,5	3,43214	3,08101	3,78481	3,38463	3,60750	3,23319

Table1: eigenvalues of the "beam mass" for different values of the rotational stiffness, mass and mass locations

2   VIBRATION OF A RESTRAINED BEAM CARRYING A POINT MASS

Consider the beam shown in Figure1, with a point mass m, restrained at the ends by translational and rotational springs.   The beam transverse displacement is $W(x,t)=w(x)sin(\omega t)$ The problem under consideration is governed by the following differential equation:

$$\frac{d^4w}{dx^4}-\beta^4 w=0 \quad \text{with} \quad \beta^4=\frac{\rho S}{EI}\omega^2 \tag{1}$$

The function w is defined in piecewise by: $w_1(y)$ and $w_2(y)$ in $]0,\varsigma[$ and $]\varsigma,1[$ respectively, with $y=\frac{x}{L}$; $\varsigma=\frac{u}{L}$. The general solution for transverse vibration in the first and second span, can be written as:

$$w_1(y)=a_1\cosh(\varsigma_i Ly)+b_1\sinh(\varsigma_i Ly)+c_1\cos(\varsigma_i Ly)+d_1\sin(\varsigma_i Ly) \tag{2}$$

$$w_2(y)=a_2\cosh(\varsigma_i L(y-\varsigma))+b_2\sinh(\varsigma_i L(y-\varsigma))+c_2\cos(\varsigma_i L(y-\varsigma))+d_2\sin(\varsigma_i L(y-\varsigma)) \tag{3}$$

In which $\varsigma_i=\sqrt[4]{\frac{\cdots S\tilde{\varsigma}_i^2}{EI}}$ for i= 1, 2, ... are the mode shape parameters of the beam with an added point mass. The constants a_j, b_j, c_j, d_j are determined by the continuity and end conditions:

At the ends: $\left.\frac{d^3w_{1i}(y)}{dy^3}\right|_{y=0}=-k_1 w_{1i}(y)\big|_{y=0}$; $\left.\frac{d^2w_{1i}(y)}{dy^2}\right|_{y=0}=k_3\left.\frac{dw_{1i}(y)}{dy}\right|_{y=0}$ (4)

Continuity conditions: $\left.\dfrac{d^3 w_{(n+1)i}(y)}{dy^3}\right|_{y=1} = k_2 w_{(n+1)i}(y)\big|_{y=1}$; $\left.\dfrac{d^2 w_{(n+1)i}(y)}{dy^2}\right|_{y=1} = -k_4 \left.\dfrac{dw_{(n+1)i}(y)}{dy}\right|_{y=1}$ (5)

$$w_{2i}(y)\big|_{y=\varsigma} = w_{1i}(y)\big|_{y=\varsigma} \quad ; \quad \left.\dfrac{dw_{2i}}{dy}\right|_{y=\varsigma} = \left.\dfrac{dw_{1i}}{dy}\right|_{y=\varsigma} \quad ; \quad \left.\dfrac{d^2 w_{2i}}{dy^2}\right|_{y=\varsigma} = \left.\dfrac{d^2 w_{1i}}{dy^2}\right|_{y=\varsigma} \qquad (6)$$

$$\left.\dfrac{d^3 w_{2i}}{dy^3}\right|_{y=\varsigma} = \left.\dfrac{d^3 w_{1i}}{dy^3}\right|_{y=\varsigma} + M(\varsigma_i L)^4 w_{1i}(y)\big|_{y=\varsigma} \qquad \text{Where } M = \dfrac{m}{\ldots SL} \qquad (7)$$

Equations 4 to 7 give a linear system with eight equations and eight unknowns whose determinant must vanish, leading via application of a Newton–Raphson algorithm, to the vibrating beam frequencies and mode shapes. The corresponding numerical results are summarised in Table1.

3   APPLICATION: A UNIFORM RESTRAINED BEAM WITH ONE POINT MASS

The effect of the added mass location on the beam first frequency, with its associated mode and curvatures, is shown in Figure 2 for various values of the rotational stiffness for M = 0.5. located at u=L/2. These results are summarised in Table 1.
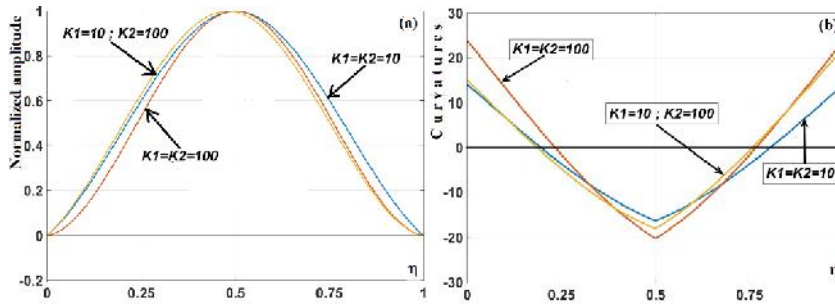

Figure. 2 (a) First mode and (b) Curvatures for M = 0.5,u=0.5 and various values of k

4   GEOMETRICALLY NONLINEAR VIBRATION OF A RESTRAINED BEAM CARRYING A CONCENTRATED MASS.

At large vibration amplitudes, the beam shown in Figure1 kinetic energy T, linear strain energy V_{lin} and nonlinear strain energy V_{Nlin} induced by large deflections can be expressed as [4]:

$$T = \frac{1}{2}\ldots S\int_0^L \left(\frac{\partial w(x,t)}{\partial t}\right)^2 dx + \frac{1}{2}m\left(\frac{\partial w(x_j,t)}{\partial t}\right)^2 ; V_{Lin} = \frac{1}{2}EI\int_0^L \left(\frac{\partial^2 w(x,t)}{\partial x^2}\right)^2 dx ; V_{Nlin} = \frac{1}{8}\frac{ES}{L}\left[\int_0^L \left(\frac{\partial w(x,t)}{\partial x}\right)^2 dx\right]^2 \quad (8)$$

Expanding $w(x,t)$ as a series of basic spatial functions: $w(x,t) = q_i(t)w_i(x) = a_i w_i \sin(\omega t)$ and applying Hamilton's principle and integrating the time functions over a period of vibration, the system dynamics is governed by [1]:

$$2[K]\{A\} + 3\left[B(\{A\})\right]\{A\} = 2\omega^2 [M]\{A\} \qquad (9)$$

in which {A} is the column vector of the basic function coefficients, and [K] and [M] are the rigidity and mass matrices, and $\left[B(\{A\})\right]$ is the nonlinear geometrical rigidity. Equation 9 is the Rayleigh-Ritz formulation of the nonlinear problem, to be solved numerically, or explicitly. From equation 9, the frequency ω may be obtained by pre multiplying the two hand sides of the equation by $\{A\}^T$, which gives:

$$\omega^2 = \frac{\{A\}^T[K]\{A\} + \dfrac{3}{2}\{A\}^T\left[B(\{A\})\right]\{A\}}{\{A\}^T[M]\{A\}} , \quad (a) \quad ; \quad \omega^2 = \frac{k}{m} + \frac{3}{2}a^2\frac{b}{m} , \quad (b) \qquad (10)$$

The single mode approach, consists of neglecting all the basic functions except a single ''resonant'' mode. Thus, it reduces equation 10(a) to 10(b), in which $[K]=k_{11}$, $[M]=m_{11}$, $[B(\{A\})]=b_{1111}$. Figure 3(a) shows the backbone curves corresponding to various values of the mass, mass location and rotational spring stiffness. Figure 3(b) shows the curvatures associated to the first nonlinear mode.
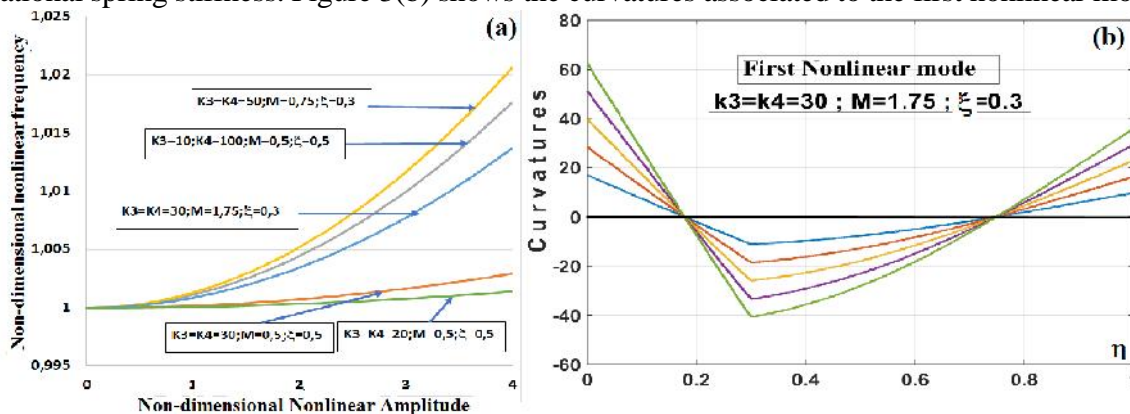

Figure 3 (a)Backbone curves for various values of the mass, mass location and rotational stiffness; (b) curvatures associated to the first nonlinear mode

5   CONCLUSION

The non-linear vibrations of a beam carrying a point mass at an arbitrary location and supported by translational and rotational springs at the two ends have been investigated analytically and a parametric study was performed, allowing examination of many combinations of classical end conditions, including elastic restraints. The dynamic equation was written at two intervals of the beam span with appropriate end and continuity conditions. After the necessary algebraic transformations, the generalised transcendental frequency equation was solved iteratively using the Newton Raphson method. Numerical results and plots have been given of the beam frequencies and first mode shape corresponding to various situations. The effect of geometrical non-linearity has then been investigated using a semi analytical method based on Hamilton's principle and spectral analysis leading to solution of a non-linear amplitude equation. A single mode approach, performed in the modal basis, has been adopted in order to obtain, for various configurations of the beam examined, the backbone curves giving the amplitude dependent nonlinear frequencies.

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