HYBRID WAVE BASED-FINITE ELEMENT UNIT CELL MODEL TO PREDICT REFLECTION, TRANSMISSION AND ABSORPTION COEFFICIENTS OF PERIODIC MATERIAL SYSTEMS

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ABSTRACT

This paper presents a hybrid unit cell method to predict sound transmission and sound absorption properties of arbitrary two-dimensional periodic structures, combining the advantages of the Wave Based Method and the Finite Element Method. The planar periodic structure, represented by its unit cell, is modelled by the Finite Element Method and the acoustic pressure field in the semi-unbounded acoustic domains is represented using the Wave Based Method. The Finite Element Method allows to include geometrical details and any combination of governing physics in the unit cell. The Wave Based Method applies dedicated approximation functions that inherently satisfy the acoustic Helmholtz equation, the Sommerfeld radiation condition and the Bloch-Floquet periodicity conditions. The dynamic fields described within both frameworks are coupled using a direct coupling strategy. The method is validated for an infinite porous plate, modelled as an equivalent fluid, with periodic, rigid, circular inclusions and is shown to be a promising tool for the analysis of complex periodic structures.

1 INTRODUCTION

Lightweight designs are emerging to save material costs and to reduce the ecological footprint of industrial products. Due to their decreased mass and retained stiffness, however, noise and vibration isolation properties are impaired, with an impact on comfort and health. Periodically structured materials, such as resonant metamaterials and porous material with inclusions are promising lightweight concepts to obtain improved low-frequency STL or absorption in a dedicated frequency band.

The vibro-acoustic performance of periodic materials is classically predicted by calculating dispersion curves, describing the wave propagation throughout the infinite periodic medium, based on a representative unit cell (UC). By comparing these dispersion curves to the dispersion curves of air, the influence on the acoustic radiation can be predicted by assessing the occurrence of acoustic coincidence.

To predict actual levels for the acoustic transmission or absorption performance, numerical simulation techniques are applied. Recently, the Transfer Matrix Method has been extended with periodicity conditions [1]. This method is very effective, but breaks down when higher order acoustic Bloch-Floquet modes have to be accounted for. The Wave Based Method (WBM) [2] and the Multipole Method [3], allow the prediction of absorption, reflection and transmission coefficients, however, yet only apply to relatively simple geometries.

To analyse the vibro-acoustic response of periodic materials consisting of arbitrarily complex UCs, this paper proposes a hybrid Finite Element - Wave Based Method Unit Cell model as an extension of the WBM [2], also towards three-dimensional applications. The dynamic fields within the bounded UC are modelled with the Finite Element Method (FEM), allowing high geometrical flexibility and arbitrary subdomains. The acoustic pressure fields inside the semi-unbounded acoustic domains are modelled with the WBM. Its approximation functions are formulated to inherently fulfil the Helmholtz equation, the Bloch-Floquet periodicity boundary conditions and the Sommerfeld radiation condition, not relying on any artificial truncation or discretization of the domain, as would be required in a pure FEM setting. The dynamic field variables of both methods are coupled at the interface, using a direct coupling approach. The method is validated for numerous cases of which one is shown in this paper and proves to be a powerful tool.

2 NUMERICAL MODEL

2.1 Mathematical problem description

An infinite, 2D periodic structure is considered, with spatial period L_x and L_y in the xy-plane, coupled to one or two semi-unbounded acoustic domains, excited by an impinging acoustic plane wave with wave number $\mathbf{k_a}$, incident at elevation θ and azimuth ψ. The periodic structure may be built up of any combination of physical subdomains for vibro-acoustic analysis. Due to the geometrical periodicity and the plane wave excitation, the resulting dynamic fields have to be periodic in the x- and y-direction. By determining the dynamic field in a single reference UC and applying the Bloch-Floquet periodicity boundary conditions, the dynamic field $\zeta(x + ML_x, y + NL_y, z)$ in any point at a distance of M and N UCs from the reference UC can be obtained:

$$\zeta(x + ML_x, y + NL_y, z) = \zeta(x, y, z)e^{-\mathrm{j}(k_{ax}NL_x + k_{ay}ML_y)},\ \forall\ M, N \in \mathbb{Z}, \tag{1}$$

where $k_{ax} = -k_a \sin\theta \cos\psi$ and $k_{ay} = -k_a \sin\theta \sin\psi$.

2.2 Hybrid Wave Based-Finite Element UC model

To analyse periodic structures of arbitrary complexity, the FEM and WBM are applied in a hybrid framework, combining best of both worlds for UC analysis. The FEM is used for the approximation of the dynamic fields within the unit cell of the periodic material as it allows high geometrical flexibility and any type of physics to be included. The WBM, on the other hand, can directly account for the semi-unbounded, periodic acoustic domain(s).

2.2.1 Finite Element Method

The FEM discretizes the considered UC of the periodic material into nodes and elements and approximates the field variables and the geometry by means of polynomial shape functions. The system of equations is obtained by approximating the governing differential equations, boundary and interface conditions via a weighted residual formulation, following a Galerkin approach. A summary of the governing equations of poro-elastic, acoustic, porous and elastic subdomains, their coupling conditions and possible boundary conditions can be found in [4].

2.2.2 Wave Based Method

The WBM, based on an indirect Trefftz approach, approximates the dynamic field variable(s) by a weighted sum of wave functions, that inherently fulfil the governing acoustic Helmholtz equation. The boundary conditions are approximated in a weak, integral sense. For this specific problem setting, to avoid integrations on infinite boundaries, semi-infinite, periodic wave functions are selected, analogous to [2] but in 3D, such that not only the acoustic Helmholtz equation is fulfilled, but also the Sommerfeld radiation condition and the Bloch-Floquet periodicity boundary conditions. All boundary conditions are thus fulfilled, except the coupling with the FE domain, which is discussed in section 2.2.3.

2.2.3 Hybrid coupling

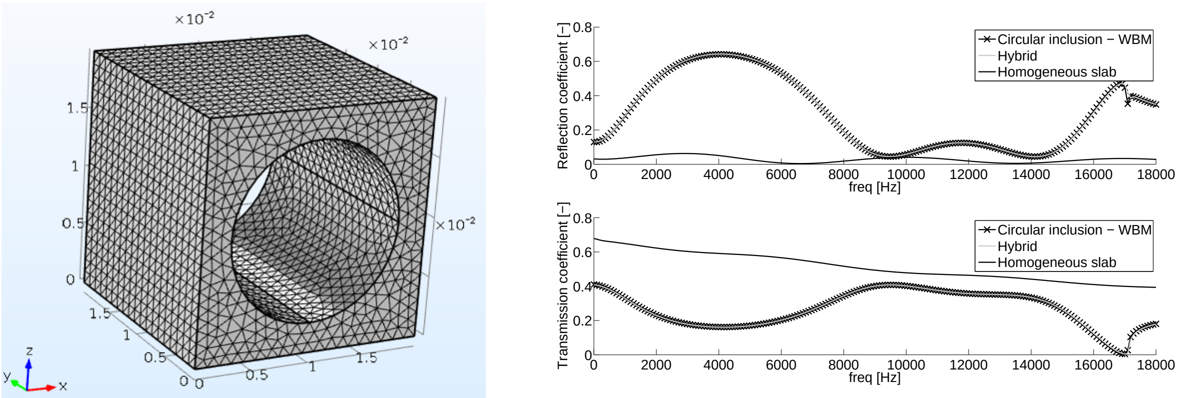
The mutual interactions between the FE and WB dynamic field variables are directly introduced into the weighted residual formulations of both models. In [4], the hybrid coupling strategies and equations are presented for different kinds of physics in the FEM part and for acoustic WBM domains and are now applied using the semi-unbounded periodic wave functions. A hybrid system of equations is obtained, coupling the degrees of freedom from both models.

2.2.4 Solution and postprocessing

Bloch-Floquet boundary conditions are applied to the hybrid system of equations for each angle and wave number of interest. The resulting system of equations is solved for the unknown nodal degrees of freedom in the FE part and the wave function contribution factors in the WB part, following a three-step procedure [4] in order to benefit from efficient solvers for sparse and dense matrix systems. Similar to [2], the reflection, transmission and absorption coefficients of the periodic structure can be obtained from the wave function contribution factors.

3 NUMERICAL VERIFICATION

To validate the hybrid UC model presented above, it is applied to analyse an infinite porous material with periodically embedded rigid circular inclusions in a transmission context and its results are compared to a WBM prediction. The geometry and material properties considered are based on the transmission case presented in [2], extruding the geometry in the third direction, such that a cubic UC with edge length $0.02m$ is obtained. The mesh and the obtained reflection and transmission coefficient are shown in Figure 1. The results obtained with the hybrid method (3D simulations) are in excellent agreement with the results obtained using the Wave Based Method (2D simulations), validating the procedure.



(a) Mesh. (b) Reflection and transmission coefficient.

Figure 1: FE Mesh of the UC and obtained reflection and transmission coefficient under normal incidence with the hybrid method and the WBM.

4 CONCLUDING REMARKS

In this work a hybrid WB-FE UC method is presented to analyze the reflection, transmission and absorption coefficient of infinite, two-dimensional periodic structures of arbitrary complexity. The method is verified by numerous validation cases of which one is presented in this paper. A promising tool is obtained to support complex lightweight periodic structure design.

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REFERENCES

[1] A. Parrinello and G.L. Ghiringhelli. Transfer matrix representation for periodic planar media. *Journal of Sound and Vibration*, 371:196 – 209, 2016.

[2] E. Deckers, C. Claeys, O. Atak, J.P. Groby, O. Dazel, and W. Desmet. A wave based method to predict the absorption, reflection and transmission coefficent of two-dimensional rigid frame porous structures with periodic inclusions. *Journal of Computational Physics*, 312:115–138, 2016.

[3] J.-P. Groby, A. Wirgin, and E. Ogam. Acoustic response of a periodic distribution of macroscopic inclusions within a rigid frame porous plate. *Waves in Random and Complex Media*, 18:409–433, 2008.

[4] S. Jonckheere. *Wave Based and Hybrid Methodologies for Vibro-acoustic Simulation with Complex Damping Treatments*. KU Leuven, PhD. thesis, 2015.