A DISCRETE MODEL FOR VIBRATION OF CRACKED BEAMS RESTING PARTIALLY ELASTIC FOUNDATIONS

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ABSTRACT

The present paper has introduced a discrete physical model to approach the problem of vibrations of cracked beams resting partially on elastic foundations. The model consisted on a beam made of several small bars, evenly spaced. The bending stiffness was modeled by spiral springs, the crack was also modeled as a spiral spring with a reduced stiffness, and the Winkler soil stiffness was modeled using linear vertical springs. Concentrated masses, presenting the inertia of the beam, were located at the bar ends. This model has the advantage of simplifying parametric studies, because of its discrete nature, allowing any modification in the mass and the stiffness matrixes. Therefore, an application for a simply supported beam resting partially on elastic foundations case is carried out.

Keywords: *Discrete; vibration; crack; foundation.*

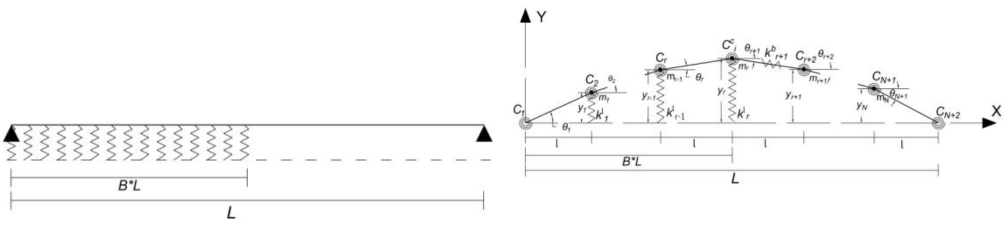
1 INTRODUCTION

The vibration of cracked beam resting on elastic foundations wears a great interest for many engineering fields such as civil and mechanical engineering. Both continuous and discrete models have been established in order to approach this problem. However, the discrete models have the advantages of being more adaptable to computer programmes. A discrete method such as the Finite Element Method (FEM) is the first to address the problem numerically [1]. The Differential Quadrature Method (DQM) is also employed for the solution to similar engineering problems involving beam vibration and foundation [2]. Therefore, P. Malekzadeh and G. Karami [3] gather the advantages of both previous methods (DQM & FEM), to investigate the free vibration and buckling analysis of thick beams on two-parameter elastic foundations.

In this paper and based on previous works [4] a discrete model for the vibration of cracked beam resting partially on elastic foundations is established. A straight application of the theory is developed, where a cracked simply supported beam is resting partially on elastic foundations.

2 GENERAL FORMULATION

Based on the model introduced by A. Khnaijar and R. Benamar [4] for nonlinear vibrations of uncracked beams resting on elastic foundation, a new model is developed here by introducing a crack beam model. The present model consists on the *N*-degree-of-freedom discrete model shown in Fig. 2, with *N* masses $m_1,\dots,m_N$, located at the ends of (*N*+1) rigid bars, connected by *(N+2-1)* spiral springs simulating the beam bending stiffness. The crack is modeled by *a* spiral spring simulating the crack reduced stiffness, estimated using the model presented in Fig. 1. The stiffness of the r^{th} spring is denoted by C_r, for $r=1$ to $(N+2-1)$), and the stiffness coefficient of the spiral spring, presenting the crack, is denoted by C^c. The bending moment M in the r^{th} spiral spring connecting the bars (*r*-1) is given by: $\mathrm{M} = -C_r\Delta\theta$; $\Delta\theta = \theta_r - \theta_{r-1}$ being the angle between the bars adjacent to the node *r*. The Winkler foundations are modeled using the longitudinal vertical spring distribution, with $k^l{}_r$ presenting the stiffness coefficient of the r^{th} linear spring, for $r=1$ to N.



(a) (b)

Figure 1. A partially supported continuous S-S beam.

The nondimensional formulation of the problem presented is essential to widen the validation basis of the results. To define the nondimensional parameters $m_{ij}, k^s{}_{ij}, k^l{}_{ij}$, let's put:

$$\frac{m_{ij}}{m^*_{ij}} = \frac{\overline{m_{ij}}}{m^*_{ij}} = \frac{\rho SL}{N+1} \; ; \; \frac{k^1{}_{ij}}{k^{1*}{}_{ij}} = \frac{\overline{k^1{}_{ij}}}{k^{1*}{}_{ij}} = \frac{EI\left(N+1\right)^3}{L^3} ; \frac{k^s{}_{ij}}{k^{s*}{}_{ij}} = \frac{\overline{k^s{}_{ij}}}{k^{s*}{}_{ij}} = \frac{EI\left(N+1\right)^3}{L^3} ;\text{for } i, j = 1,..,N \quad (1)$$

As the nondimensional formulation is established, the general expression for $m^*_{ij}, k^{*s}{}_{ij}$ and $k^{*l}{}_{ij}$ for the discrete model become as follows:

- The nondimensional Mass tensor $\left[\mathbf{M}^*\right]/m^*_{ij}$:

$$m^*_{ij} = \delta_{ij} \text{ for } i, j = 1,.., N \tag{2}$$

- The nondimensional Spiral springs tensor $\left[\mathbf{K}^{s*}\right]/k^{s*}_{ij}$

$$k^{*s}_{(r-2)r} = \left(\frac{I_r^x}{I}\right) \text{ for } r = 3,..., N \tag{3}$$

$$k^{*s}_{(r-1)r} = -\frac{2}{I}\left(I_r^x + I_{r+1}^x\right) \text{ for } r = 2,..., N \tag{4}$$

$$k^{*s}_{rr} = \frac{1}{I}\left(I_r^x + 4I_{r+1}^x + I_{r+2}^x\right) \text{ for } r = 1,..., N \tag{5}$$

and the other values of k^{*s}_{ij} are obtained by symmetry relations, or are equal to zero.

$$I_i^{cr} = \frac{bh^3(1-\xi)^3}{12} \text{ with } \xi = \left(\frac{a}{h}\right) \text{ and } I = \frac{bh^3}{12} \tag{6}$$

I_{cr} and I,for rectangular sections, are respectively being the inertial moment of the reduced beam, calculated by putting the neutral fiber of this section in the center of the reduced beam section and the inertial moment for uncracked section, and ξ refers to the dimensionless crack depth ratio.

- The nondimensional Winkler springs tensor $\left[\mathbf{K}^{*l}\right]/k^{*l}_{ij}$

$$k^{*l}_{ij} = \alpha\delta_{ij} \text{ for } i, j = 1,..., N \tag{7}$$

The Winkler stiffness coefficient is given by:

α and λ being the nondimensional parameter:

$$\alpha = \frac{kL^4}{EI\left(N+1\right)^4} = \frac{\lambda}{\left(N+1\right)^4} \text{ with } \lambda = \frac{kL^4}{EI} \tag{8}$$

3 APPLICATION: A PARTIALLY SUPPORTED BEAM WITH A SIMPLY SUPPORTED ENDS

A simply supported beam is assumed to be subjected to the effect of partial intermediate supports Fig. 1. Fig. 2 shows the cracked beam first two modes frequencies, for a simply supported beam, versus the crack position (*c/l*) and depth (ξ=*a/h*), with no elastic foundation (*B*=0). Fig. 3 shows the cracked beam first two modes frequency, for a simply supported beam, versus the crack position (*c/l*) and depth (ξ=*a/h*), the ratio of the supported span to the total span *B* chosen covers 50 percent of its span (*B*=0.5) and the soil stiffness $\lambda = \pi^4$. The shift in the curves for the case with elastic soil proves the effect of the soil stiffness and position on the beam frequencies.

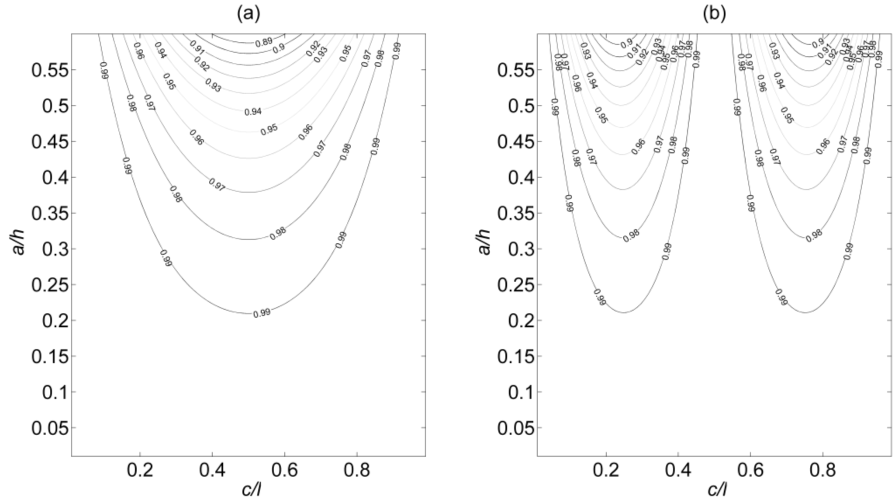


Figure 2. The cracked beam frequency, for a simply supported beam, versus the crack position (*c/l*) and depth (*ξ=a/h*) for the first 2 modes: *B*=0.

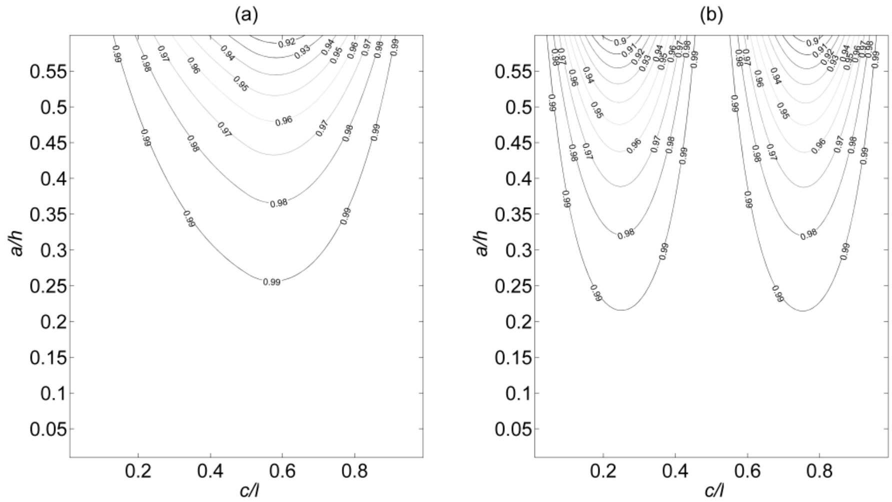


Figure 3. The cracked beam frequency, for a simply supported beam, versus the crack position (*c/l*) and depth (*ξ=a/h*) for the first 2 modes: *B*=0.5 and $\lambda = \pi^4$

4 CONCLUSION

A discrete model of cracked beam resting partially on elastic foundation was introduced in the present paper. An application was carried out for a simply supported beam, where a frequency curves versus crack magnitudes and locations were drawn for a partially supported cracked beam.

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