

# IDENTIFICATION OF STRUCTURAL FORCES FROM ACOUSTIC MEASUREMENT USING THE INVERSE SIMPLIFIED ENERGY METHOD

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# ABSTRACT

The identification of structural forces acting on plate coupled with an acoustic cavity from an acoustic measurement is presented in this paper. An energy-based method called simplified energy method (MES) has been presented to predict the radiation of the plate coupled with an acoustic cavity at high frequency range. This paper proposes to use this energy approach to solve inverse structural problems to identify the structural sources thanks to the inverse formulation of the method (IMES). Example concerning different acoustic measurement points are presented to validate the proposed method.

### **1 INTRODUCTION**

The identification of structural forces acting on structures from operating measurements is an important topic that has been treated by several researchers. Nevertheless, direct measurements of sources are not feasible to quantify the position of the exciting forces. As a result, an inverse process for estimating the exciting sources is often employed to solve the problem. In the range of medium and high frequencies numerical methods as the finite element method (FEM) and the boundary element method (BEM) present limits when the frequency is increased. For that, the energy methods based on energy quantities are often used. Among these methods the simplified energy method (MES). The direct theory formulation has been applied in various domains including beam [1], membrane and plates [2] and acoustic applications [3]. An inverse energy flow method (IMES) has been developed for acoustical application [4] and plate [5], and for a complex structure modeled with a set of assembled plates [6]. The main novelty of this paper is to develop this inverse method in order to detect the structural force applied in plate coupled with an acoustic cavity through a measurement data of the acoustic energy field.

### 2 DIRECT MES FORMULATION

The simplified energy method is a vibro-acoustic method developed for the purpose to predict the energy density distribution for structural acoustic problem in high frequency ranges. This method is based on a description of two local energy quantities: the energy density W is defined as a sum of the kinetic and potential energy densities and the energy vector  $\vec{I}$  which is the energy flow inside systems. The energy  $W_s$  on the structure is defined as the sum of the primary source  $\rho_s$  (direct field) coming from the excitation point S, and the fictitious sources  $\sigma_s$  coming from the boundary P of structure as shown in Figure 1.



Figure 1. System decomposition.

$$W_s(M_s) = \int_{\Omega_s} \varrho_s(S) G_s(S, M_s) dS + \int_{\partial \Omega_s} \sigma_s(P) \overrightarrow{u}_{SM_s} \cdot \overrightarrow{n}_P G_s(P, M_s) dP \tag{1}$$

The acoustical energy  $W_a$  radiated from the structure is located at the excitation points S and structure boundaries P. In addition, according to the assumption that propagative waves are uncorrelated, the acoustical energy  $W_a$  at any point  $M_a$  inside the cavity is the sum of energy radiated from the excitation sources  $W_a^{exci}$  and energy radiated from the structure extremity  $W_a^{edge}$ :

$$W_a(M_a) = W_a^{exci}(M_a) + W_a^{edge}(M_a)$$
<sup>(2)</sup>

where

$$W_a^{exci}(M_a) = \int_{\Omega_s} \varrho_a(S, \overrightarrow{u}_{SM_a}) G_a(S, M_a) dS,$$
(3)

and

$$W_a^{edge}(M_a) = \int_{\Omega_s} \sigma_a(P, \overrightarrow{u}_{PM_a}) G_a(P, M_a) dP.$$
(4)

#### **3** INVERSE MES FORMULATION

This section focuses on developing an inverse MES formulation for a plate coupled with an acoustic cavity, by applying a discrete format of the acoustic energy equation. The IMES formulation aims to invert the matrix formulation of Equation. (2). The structural force applied in plate can then be estimated and localized from an acoustic measurement. It is expressed as follows:

$$P^{struc} = S_a^+ \cdot W_a \tag{5}$$

where + is the pseudo inverse,  $W_a = W_a^{exci} + W_a^{edge}$  and  $S_a = S_a^{exci} + S_a^{edge}$ . Matrix  $S_a$  can be well-conditioned, but is often ill-conditioned, which will disturb the results. In the next parts, we studied the matrix  $S_a$  inversion influence on the numerical results.

## **4 NUMERICAL RESULTS**

This section deals with numerical tests of different cases in order to validate the presented formulation. The presented system is a structure which consists of a plate ( $\rho_s = 7800 \text{ kg/m}^3$ ,  $\nu$ 



Figure 2. Boundary conditions.

= 0.3,  $E = 210.10^9$  Pa) coupled with an acoustic cavity ( $\rho_a = 1.3 \text{ kg/m}^3$ ,  $c_a = 340 \text{ m/s}$ ). The structure is a rectangular plate, their lengths are L = 1 m, l = 0.8 m and thickness  $h = 3.10^3$  m, clamped on their extremity and damped with coefficient  $\eta_s = 1$  %. The acoustic cavity is undamped and with dimensions (L = 1 m, l = 0.8 m, H = 0.3 m) as shown in Figure 2. The plate is excited with an input power  $P_{in} = 1.551$  W/m<sup>2</sup>, at point S given by (0.45 m, 0.45 m, 0). The numerical simulation methodology presented in Figure 2 consists of implementing the inverse energy flow approach IMES when using a set of density energy prediction based in the finite element method FEM modelled using the finite element software COMSOL multiphysics (FEM/IMES simulation). This example deals with the influence of acoustic measurements in the detection and quantification of the vibration source. A new numerical methodology to identify the loads acting on the structure using FE method was presented in this example. A more realistic test case was then considered. The first step consists in subdividing a plane



Figure 3. Flow chart of numerical methodology.

parallel to the plate with a distance e = 0.05 m, into nine areas, and placing a microphone in the middle of each area. In fact, the  $n_{M_a} = 9$  acoustic measurement were distributed as shown in Figure 4.



Figure 4. FEM/IMES Simulation for  $d_x = 0.33$  m and  $d_y = 0.26$  m

A color map of the estimated power repartition in plate was drown in Figure 4. From a first view, the highest value of power was estimated with a value equal to  $0.8949 \text{ W/m}^2$ . The next step consists in distributing the same number of microphone near the highest power surfaces with an equal distance  $d_x = d_y = 0.09$  m as shown in Figure 5. As seen in this figure, a good input power prediction is observed. The results show that the amount of source information increases as the measurement point is so close from the source. Therefore, it is preferable that the distances between source and nearest measurement points are as short as possible. Moreover, it seems that choosing a good disposition of acoustic measurement is quite important to estimate the input force. Finally, this section confirmed that the present method tends to estimate the input force in the structure for a few number of acoustic measurements, and can be applied for industrial problems, such as aeronautics and automotive industry.



Figure 5. FEM/IMES Simulation for  $d_x = d_y = 0.09$  m.

# **5** CONCLUSION

In this paper, numerical results are presented in order to localize the vibration source from acoustic measurement. The new IMES formulation based predictive tool has been first developed. Several test cases involving different measurement points and external forces were considered. The inverse solutions were compared to results given by the direct MES and FEM simulations. The obtained results confirm that the proposed approach exactly estimates the excitation force with a low density of measurement points.

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