GEOMETRICALLY NONLINEAR OF ORTHOTROPIC PLATES USING SEMI-ANALYTICAL METHOD

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ABSTRACT

The present paper concerns the nonlinear dynamic behaviour of orthotropic rectangular plate under boundary conditions (C-C-C-SS) and (C-C-SS-SS). The main objective is to find semi analytical solutions for the first non-linear mode shapes and the associated non-linear frequencies of the composite plates at large vibration amplitudes. The basic formulation of nonlinear free vibrations has been developed based on the classical plate theory (CPT) and the nonlinear strain-displacement relation. The nonlinear governing equations are derived from Hamilton's principle and the Von Kármán geometrical non-linearity assumptions. Assuming the out-of-plane displacement as a double trigonometric function, the in-plane displacement components are found by solving the nonlinear algebraic equations of motion expressed in terms of displacements. The improved version of the Newton-Raphson method and the semi-analytical model developed by El Kadiri et al. for fully clamped rectangular plates, has been adapted to the above cases.

1 INTRODUCTION

Laminated composite plates are frequently used in various engineering applications in the aerospace, mechanical, marine, and automotive industries because of their advantages such as high stiffness-to-weight and strength-to-weight ratios. In the case where these structures are subjected to dynamic loads may induce large amplitude vibrations and, thus, the structure may exhibit significant nonlinear behaviour that must be studied for the efficient design of such structures.
Numerous methods have been developed to perform geometrically nonlinear analysis of plates. Benamar et al [1] presented a theoretical formulation of the plate vibration problem at large displacement amplitudes. Han and Petyt [2], Ribeiro and Petyt [3] have been presented dealing with the geometrically non-linear dynamic behaviour of symmetrically laminated plates by using the hierarchical finite element method (HFEM). Harras and Benamar [4] investigated theoretical and experimental of the non-linear behaviour of various fully clamped rectangular composite panels at large vibration amplitudes. El Kadiri et all [5, 6] presented a semi-analytical method, based on Hamilton's principle and spectral analysis, for the determination of the geometrically non-linear free response of thin straight structures. Several review articles on orthtropic plates have been reported in the literature by various researchers, such as Leissa [7], Reddy [8], and Noor et al. [9].
In the present paper the method developed by El Kadiri et al. is extended to the geometrically nonlinear analysis of orthotropic plate with two boundary conditions (C-C-C-SS) and (C-C-SS-SS). This boundary conditions are widely used in aerospace structures. On the other hand, this study will contribute to generalize and extend the model to different conditions.

2 THEORY

Consider the transverse vibration of C-C-SS-SS rectangular plate which is clamped on two edges, and simply supported in the other edges. This plate is shown in figure 1.

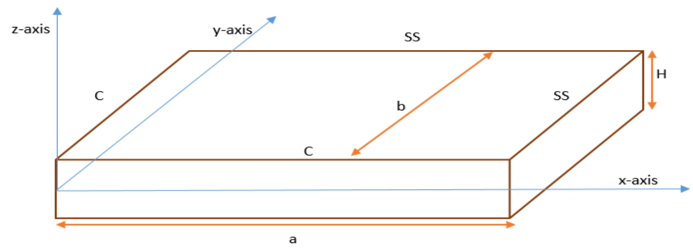

Figure 1. Plate notation

For the classical plate laminated theory, the strain-displacement relationship for large deflections:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_{y_y} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial U}{\partial x} \\ \dfrac{\partial V}{\partial y} \\ \dfrac{\partial U}{\partial x} + \dfrac{\partial V}{\partial y} \end{bmatrix} + \begin{bmatrix} -\dfrac{\partial^2 W}{\partial x^2} \\ -\dfrac{\partial^2 W}{\partial y^2} \\ -2\dfrac{\partial^2 W}{\partial x \partial y} \end{bmatrix} + \begin{bmatrix} \dfrac{1}{2}\left(\dfrac{\partial W}{\partial x}\right)^2 \\ \dfrac{1}{2}\left(\dfrac{\partial W}{\partial y}\right)^2 \\ \dfrac{\partial W}{\partial x}\dfrac{\partial W}{\partial y} \end{bmatrix} \equiv \{\varepsilon\} = \{\varepsilon^0\} + z\{\kappa\} + \{\lambda^0\} \qquad (1)$$

where $\{\varepsilon^0\}$ and $z\{\kappa\}$ are the membrane and the flexural strain tensors, respectively, and U, V, W are the middle surface displacement components in the x, y and z directions respectively.
The free vibrations of the structure are governed by Hamilton's principle which is symbolically written as

$$\delta \int_0^{2\pi} (V - T)dt = 0 \qquad (2)$$

In which δ indicates the variation of the integral. V and T are respectively the total strain energy and the kinetic energy, where $V = V_a + V_b$. Replacing T and V in this equation by their expressions given above, integrating the time functions, and calculating the derivatives with respect to the a_i, leads to the following set of non-linear algebraic equations:

$$3a_i a_j a_k b^*_{ijkl} + 2a_j k^*_{ir} - 2a_i \omega^{*2} m^*_{ir} = 0, \quad i = 1, \dots, n. \tag{3}$$

$$m_{ij} = \rho H^5 ab\, m^*_{ij}, k_{ij} = \frac{aH^5E}{b^3} k^*_{ij}, b_{ijkl} = \frac{aH^5E}{b^3} b^*_{ijkl} \tag{4}$$

a, b: length, width of the plate; E: Young's modulus; H: plate thickness; a_k: contributions corresponding to the kth basic functions; ρ: mass density per unit volume of the plate.
ω, and $\omega^*$are the frequency and non-dimensional frequency parameters respectively.
k^*_{ij}, $m^*_{ij}$and b^*_{ijkl}: General term of the non-dimensional rigidity tensor, mass tensor and non-linearity tensor respectively.

2.1 Explicit procedure

In modal functions basis for the first mode (MFB):

$$w^*(x^*, y^*) = \sum_{i=1}^{n} a_i \Phi^*_i(x^*, y^*) = \{A\}^T\{\Phi^*\} \tag{5}$$

with $\{\Phi^*\}^T = [\Phi^*_1 \;\; \Phi^*_2 \dots \Phi^*_n]$ and $\{A\}^T = [a_1\, \epsilon_2 \; \dots \epsilon_n]$

$$\epsilon_r = \frac{3a_1 b^*_{r111}}{2((k^*_{11} + a_1^2 b^*_{1111})\frac{m^*_{rr}}{m_{11}} - k^*_{rr})} \quad (r = 2,\, 3 \dots 16) \tag{6}$$

$$w^*_{nl1}(x^*, y^*, a_1) = a_1\Phi^*_1(x^*, y^*) + \epsilon_2\Phi^*_1(x^*, y^*) + \dots + \epsilon_{16}\Phi^*_{16}(x^*, y^*) \tag{7}$$

The chosen basic functions $P^*_i(x)$ were the linear clamped-simply supported beam functions and $Q^*_i(x)$ were linear clamped-clamped beam.

- Clamped-Simply supported beam

$$P^*_i(x) = ch\,(l_i x^*) - \cos(l_i x^*) - (sh\,(l_i x^*) - \sin\,(l_i x^*))\,(\frac{(ch\,(l_i) - \cos\,(l_i)}{sh\,(l_i) - \sin l_i)}) \tag{8}$$

- Clamped-Clamped beam

$$Q^*_i(x) = \frac{ch\left(\frac{\upsilon_i x}{a}\right) - \cos(\frac{\upsilon_i x}{a})}{ch\,(\upsilon_i) - \cos\,(\upsilon_i)} - \frac{sh\left(\frac{\upsilon_i x}{a}\right) - \sin\,(\frac{\upsilon_i x}{a})}{sh\,(\upsilon_i) - \sin\,(\upsilon_i)} \tag{9}$$

3 RESULTS OF NON-LINEAR ANALYSIS

The geometrical and material properties are defined in Table 1.

Geometric properties	Material properties
Orientation of principale axes :$[90,45,-45,0]_{sym}$ a=485.7mm ; b=322.9 mm ; h=1 mm	Ex=120.5 GPa; Ey=9.63GPa; Gxy=3.58 GPa; ν_{xy}=0.32;$\rho = 1540\ kg/m^3$

Table 1. Geometric and material properties of thin plate

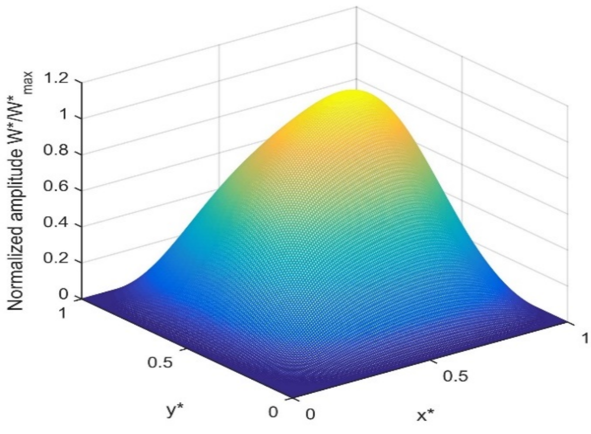

Figure 1. First non-linear mode rectangular C-C-SS-SS plate α=b/a=2/3, w*(x*,y*).

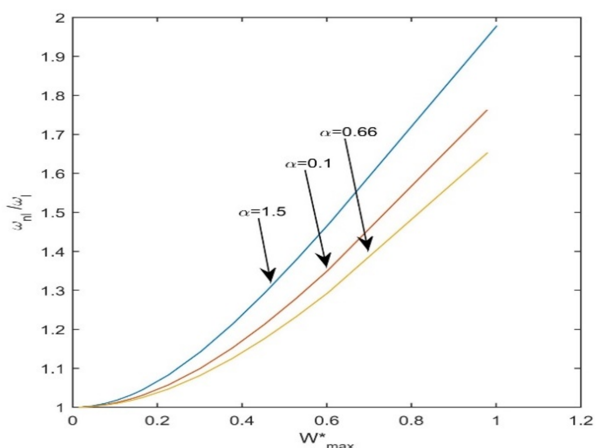

Figure 2. Comparison of the change frequency of the first mode for: $\alpha =$ 1,5 ; $\alpha = 1$; $\alpha = 1{,}5$.

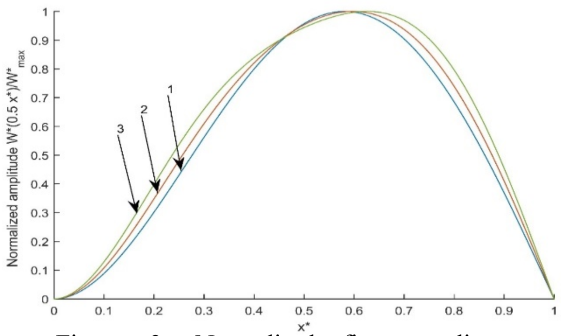

Figure 3. Normalised first non-linear mode rectangular C-C-SS-SS plate α=1,5, x*=0,5. Curve 1, lowest amplitude ; curve 3, highest amplitude.

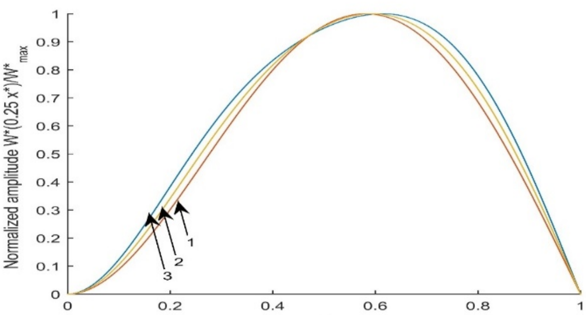

Figure 4. Normalised first non-linear mode rectangular C-C-SS-SS plate α=1,5, x* =0,25. Curve 1, lowest amplitude ; curve 3, highest amplitude.

Comparison of the non-linear frequency and linear frequency of the C-C-SS-SS rectangular plate, for various plate aspect ratios (α =a/b), where a_1 represent the amplitude of vibration (Table 2).

α =a/b	0.4	0.66	1	1.5
ω_l^*	79.420	86.249	102.37	143.73
$\omega_{nl}^*(a_1 = 0{,}01)$	79.4366	86.268	102.40	143.79
$\omega_{nl}^*(a_1 = 0{,}25)$	88.988	97.063	117.93	174.59

Table 2. Comparison of non-dimensional frequency parameters

4 CONCLUSION

- The first non-linear mode of C-C-S-S and the explicit analytical expressions for the higher mode contribution coefficients to the first non-linear mode shape have been obtained.
- Numerical results obtained from the application of C-C-SS-SS rectangular plate with different values of aspect ratio $\boldsymbol{\alpha}$ have been given.
- The validity of the current approach will be compared later those of finite element methods (FEM),

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