STATISTICAL ENERGY ANALYSIS, ASSUMPTIONS AND VALIDITY

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ABSTRACT

This study outlines the question of validity of statistical energy analysis with regard to its assumptions. We discuss the necessity of four assumptions: rain-on-the-roof excitation, weak coupling, large number of modes and light damping. We show that when all of these assumptions are satisfied, statistical energy analysis provides a satisfactory result but when one of these assumptions is violated, statistical energy analysis prediction presents a discrepancy compared to a reference calculation. The discussion is illustrated with a simple example of coupled plates.

1 INTRODUCTION

Statistical energy analysis [1, 2] is a well-known theory of sound and vibration suited to the domain where the number of modes is so high that the usage of finite element method is not tractable. Statistical energy analysis is based on statistical physics concepts such as mean-free path, modal density and equipartition of energy. It is the counterpart of Sabine's theory of reverberation in room acoustics. Although it is commonly claimed that statistical energy analysis applies at high frequencies, it is more exact to say that statistical energy analysis is the theory of thermal equilibrium where sound and vibration are diffuse.

2 STATISTICAL ENERGY ANALYSIS

The principle of statistical energy analysis is quite simple. The main result is Lyon's law [3], or coupling power proportionality, which states that two subsystems in which the vibrational field (velocity field for structure or acoustical pressure for sound) is diffuse and lightly coupled exchange a vibrational power proportional to the difference of their modal energies. This reads

$$P_{ij} = \omega \eta_{ij} n_i \left(\frac{E_i}{n_i} - \frac{E_j}{n_j} \right) \tag{1}$$

where P_{ij} is the mean power between subsystems i and j, E_i the mean energy, n_i the modal density, ω the centre of the frequency band of analysis, and η_{ij} the coupling loss factor. If the two subsystems are coupled through a spring of stiffness K then the coupling loss factor is

$$\eta_{ij} = \frac{\pi K^2 n_j}{2\omega^3 M_i M_j} \tag{2}$$

where M_i is the total mass of subsystem i. If the two subsystems are two-dimensional (like plates) and are coupled through a line of length L whose mean transmission efficiency is noted T, then

$$\eta_{ij} = \frac{L c_{g_i} T}{\pi \omega S_i} \tag{3}$$

where c_{g_i} is the group speed of waves in subsystem i and S_i its area. In all cases, the coupling between subsystems must be conservative.

To derive this result, the minimal list of assumptions is

- Rain-on-the-roof excitation
- Light damping
- Large number of modes
- Weak coupling

When these conditions are satisfied, the vibrational field is diffuse in all subsystems or equivalently, equipartition of energy is reached. Each subsystem is therefore in the state of thermal equilibrium. The weak coupling limits the exchange of energy to a small level which does not disturb the diffuse field in the vicinity of coupling.

3 DIFFUSE FIELD

The first three conditions ensure that a diffuse field is established in all subsystems [4]. In Figure 1 is shown the relative standard deviation of repartition of energy in rectangular plates. The abscissa is the number of wavelengths per mean-free-path and the ordinate is the damping loss factor of the plate. We see that the domain of diffuse field for which the standard deviation is small is confined by two criteria. The frequency must be high (vertical line) and the attenuation of waves must be small (horizontal line tilted on the right).

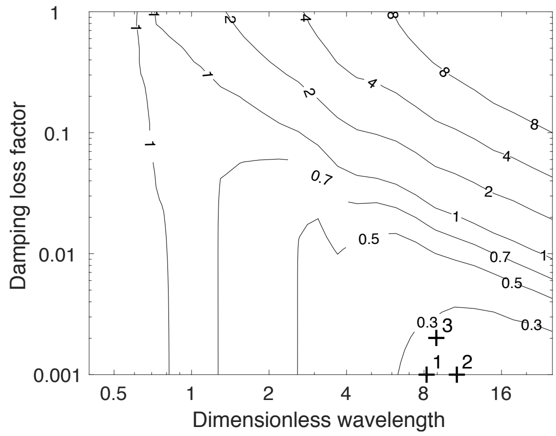

Figure 1: Relative standard deviation of repartition of energy in rectangular plate in the dimensionless wavelength - damping loss factor plane. The zone of diffuse field is confined to high frequencies and small damping.

4 EXAMPLES OF COUPLED PLATES

The first example is shown in Figure 2. The structure is made of six rectangular plates assembled by right angle couplings. Plate 1 is submitted to a random transverse force field (rain-on-the-roof). The response is observed in plate 6. Three calculations are performed: a reference calculation based on a closed form solution of the governing equations, a SEA calculation with Equations (1) and (3), and a geometrical acoustics prediction (see Reference [5]). We observe that when the damping is light ($\eta = 1\%$), the prediction of SEA is always correct. The four octave bands are located in the region of diffuse field of Figure 1. The error of SEA, compared with a reference calculation is negligible. But when the damping is strong ($\eta = 10\%$), significant errors of SEA appear. Note that geometrical acoustics prediction (ray) is still valid because the frequency remains high. This examples highlights that SEA requires two conditions in general: large number of modes (high frequency) and low damping.

The second example is shown in Figure 3. The system is made of three rectangular plates with random resonators and coupled through a spring of stiffness K. The coupling strength is controlled by varying K. A single point random force assumed to be white noise is applied to the top plate. Two calculations are performed: SEA by applying Equations (1) and (2) and a reference calculation by a semi-analytical method. We observe that the thermal conductivity $\beta_{\mathrm{SEA}} = \omega\eta_{ij}n_i$ predicted by SEA is correct compared to the ratio $\beta_{\mathrm{REF}} = P_{ij}/(E_i/n_i - E_j/n_j)$ computed by the reference calculation when the coupling is weak. But when the coupling strength increases, a large discrepancy is observed between SEA and reference. It may even arise that the flow of energy is reversed giving a negative value of β. This anti-thermodynamics flow of energy has been observed even for a population of nominally similar plates with different realisations of resonators as shown by the grey zones of Figure 3.

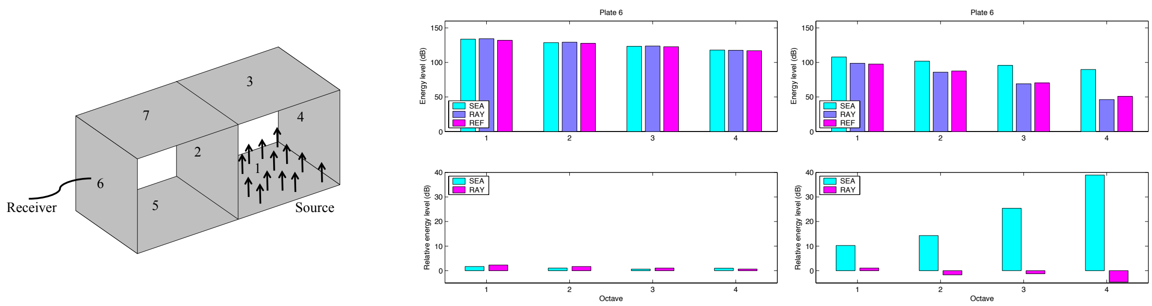

Figure 2: Six rectangular plates in bending vibration and results for $\eta = 1\%$ (middle) and $\eta = 10\%$ (right). The upper bar diagrams give the energy versus octave band by three methods: SEA, ray and reference calculation. The lower bar diagrams give the error of SEA and ray-tracing compared to reference.

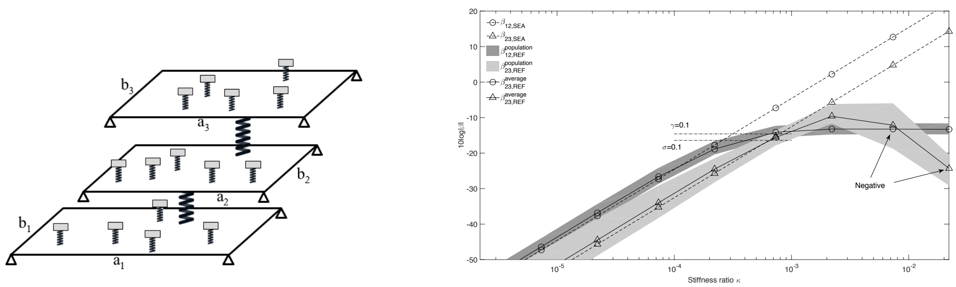

Figure 3. Three rectangular plates in bending vibration with random resonators.

5 CONCLUDING REMARKS

In this paper, we have shown that statistical energy analysis is based on several assumptions that are random excitation, light damping, large number of modes, and weak coupling. Although the first three conditions may be reduced to the single condition of diffuse field in all subsystems, the last one is an imperative requirement that cannot be relaxed in general.

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