ON THE STABILITY LOSS OF LIMIT CYCLE OSCILLATIONS NEAR STRONG RESONANCES: SYNCHRONIZATION AND HETEROCLINIC BIFURCATION

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ABSTRACT

In self-excited nonlinear oscillators subjected to harmonic forcing, frequency-locking can occur near strong resonances. This phenomenon results from synchronization between the frequency of the forcing and the frequency of the limit cycle oscillation leading to frequency-locked motions for which the response of the system follows the forcing frequency.

In the case of 1:4 resonance, which is considered as one of the unsolved problem in nonlinear dynamics [1], the limit cycle loses its stability at the synchronization via heteroclinic bifurcation. Usually, the transition between quasi-periodic and synchronized motions occurs via heteroclinic connections at two different frequencies causing hysteresis and bistability. Therefore, analytical approximation of heteroclinic bifurcations near the 1:4 resonance is of great importance since they determine the locations at which the frequency-locked motion takes place.

The existence of heteroclinic orbits in ordinary differential equations corresponds to the existence of coherent structures such as solitons and fronts in certain partial differential equations. For instance, they form the profiles of traveling wave solutions in reaction–diffusion problems and spatially localized post-buckling states in static dynamics. Also, heteroclinic orbits correspond to the onset of various types of synchronization in certain problems in physics and biology [2,3]. Therefore, one of the challenging problems is the analytical capture of the heteroclinic bifurcations location near the strong resonances [1]. To the best of our knowledge, rigorous analytical

expressions of heteroclinic bifurcation near these resonances have not been obtained, only numerical methods have been performed [4,5]. In this talk, recent analytical methods to capture approximation of such heteroclinic bifurcations in the problem of stability loss of limit cycle oscillations near the 1:4 resonance will be presented. The problem of 1:3 resonance will be also discussed.

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