A FIXED POINT ALGORITHM AND MODEL REDUCTION IN JOINTED STRUCTURES SIMULATION

N. Peyret[1], G. Chevallier[2], A. Meurdefroid[1]

[1]Laboratoire Quartz
Supméca, 3, rue Fernand Hainaut, F-93400 Saint-Ouen, FRANCE
Email: nicolas.peyret@supmeca.fr, anthony.meurdefroid@supmeca.fr

[2]FEMTO-ST Institute Department of Applied Mechanics
Univ. Bourgogne Franche-Comté, 24, rue de l'épitaphe, F-25000 Besançon, France
gael.chevallier@femto-st.fr

ABSTRACT

The bolted joints have a strong impact on the damping and the stiffness of the structures. This impact remains difficult to predict because of the difference between the length scale of the real contact area and the wavelength of the vibration modes, and the uncertainties on the real geometry of the contact area. The method proposed in this paper is to divide the jointed structure into two parts : the linear part (L) and the non-linear one (NL) located around the joint. First, a linear analysis is performed on the global structure, neglecting dissipation inside the joint, to determine the normal modes of the structure. The normal modes subspace is normalized to the sti_ness matrix to associate to each eigenvectors the same strain energy. In the neighborhood of the bolted joint, eigenmodes are not orthogonal to each other's. Thus, it is possible to reduce the size of the subspace spanned by the local eigenmodes. Moreover, most of them do not dissipate energy. Thus, it is possible to select the only ones that inuence the joint behaviour. We introduce the Principal Joint Strain Basis (PJSB) which is the optimal Ritz basis deduced from the structure eigenmodes, and simplified thanks to the analysis of the dissipation potential of each eigenmode.
The dissipation potential is estimated by the energy coupling in the joint computed from the sensitivity of the eigenfrequency to the tightening configuration, i.e. when the surfaces of the interface are tied or when the tightening is very low. Then, we assume that a metamodel is able to represent the behaviour of the joints. In order to build it, we apply the PJSB as a loading on a finite element model of the joint and we post-process the results in order to use them in a reduce order model.

1 INTRODUCTION

The bolted joints strongly influence both the damping and the stiffness of the structures. Unfortunately, they remain difficult to predict, mainly because the physics involved occur with very different scales, i.e. the length scale of the real contact areas and the wavelength of the vibration modes. This make the classical finite element method difficult to use because in the contact areas, the mesh have to be fine whereas it can be coarse elsewhere. To overcome these difficulties, the purpose of this paper is to use a spatial decomposition of the domain in two subdomains that allows working with different spatial dicretizations on both subdomains. The subdomains are ΩS that includes all the structure parts and ΩJ that includes the joints. The first idea, detailed in section 2, is to solve alternatively the problems written on ΩS and ΩJ until the results on both become similar. This is strongly inspired of the LATIN method [6] but adapted to study periodic vibrations using Harmonic Balance Method (HBM) coupled with the Alternating Frequency Time (AFT) algorithm [2] or more recently [3]. The second idea of this work is to reduce the order of the model by using Ritz basis and meta-models spanning the subspace that contains the solutions.

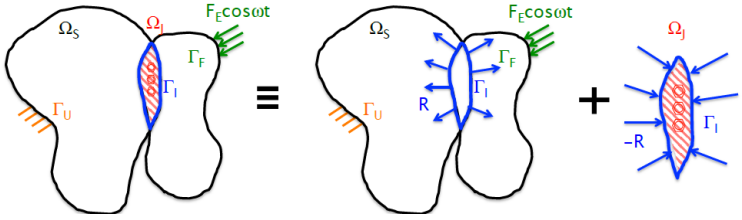

Figure 1. Typical schema for the study of jointed structures vibrations. Subdomain decomposition of the whole problem

2 DECOMPOSITION OF THE PROBLEM

This work focuses on dynamics of structural assemblies that involve parts that behave linearly and bolted joints that behave nonlinearly. Overall, the vibrations of the structure can be studied by solving the following problem:

$$\mathbf{M\ddot{U}} + \mathbf{KU} + \mathbf{F}(\mathbf{U}, \dot{\mathbf{U}}) = \mathbf{F_E} \tag{1}$$

Where U is the displacement field, F_E is the excitation force and F is the non linear force induced by friction in the joints. M, K are the mass and the stiffness matrices. Equation 1 can be detailed by separating the Degrees of Freedom that belong to each subdomain Ω_S and Ω_J and to the interface Γ_I.

$$\begin{bmatrix} \mathbf{M}^S_{LL} & \mathbf{M}^S_{LI} & \mathbf{0} \\ \mathbf{M}^S_{IL} & \mathbf{M}^S_{II}+\mathbf{M}^J_{II} & \mathbf{M}^J_{IL} \\ \mathbf{0} & \mathbf{M}^J_{LI} & \mathbf{M}^J_{LL} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{U}}^S_L \\ \ddot{\mathbf{U}}_I \\ \ddot{\mathbf{U}}^J_L \end{bmatrix} + \begin{bmatrix} \mathbf{K}^S_{LL} & \mathbf{K}^S_{LI} & \mathbf{0} \\ \mathbf{K}^S_{IL} & \mathbf{K}^S_{II}+\mathbf{K}^J_{II} & \mathbf{K}^J_{IL} \\ \mathbf{0} & \mathbf{K}^J_{LI} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U}^S_L \\ \mathbf{U}_I \\ \mathbf{U}^J_L \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{F}^J(\mathbf{U}^J_L, \dot{\mathbf{U}}^J_L) \end{bmatrix} = \begin{bmatrix} \mathbf{F}^S_E \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{2}$$

In this work, we focus on harmonic excitation. For such kind of problem, the HBM is often used as it allows to compute steady-state periodic vibrations efficiently. Assuming steady-state vibrations and a periodic response, **U** can be written as:

$$\mathbf{U} = \mathbf{U}^0 + \sum_{h=1}^{H} \frac{\mathbf{U}^h}{2} e^{jh\omega t} + \frac{\mathbf{U}^{h*}}{2} e^{-jh\omega t} \tag{3}$$

in a basis of exponential functions. Assuming the periodicity of the non-linear force, we can also expand it on a Fourier Basis

$$\mathbf{F}(\mathbf{U}, \dot{\mathbf{U}}) = \mathbf{F}^0 + \sum_{h=1}^{H} \frac{\mathbf{F}^h}{2} e^{jh\omega t} + \frac{\mathbf{F}^{h*}}{2} e^{-jh\omega t} \tag{4}$$

Balancing, according to their harmonic order, each term of the obtained equation, we get:

$$\begin{cases} [-\omega^2\mathbf{M}+\mathbf{K}]\,\mathbf{U}^1+\mathbf{F}^1(\mathbf{U}^1\ldots\mathbf{U}^H)=\mathbf{F_E} \\ \cdots \\ [-H^2\omega^2\mathbf{M}+\mathbf{K}]\,\mathbf{U}^H+\mathbf{F}^H(\mathbf{U}^1\ldots\mathbf{U}^H)=\mathbf{0} \end{cases} \tag{5}$$

This can be summarized in a more compact form:

$$[\Lambda]\,\hat{\mathbf{U}}+\hat{\mathbf{F}}=\hat{\mathbf{F}}_\mathbf{E} \tag{6}$$

Where Λ is the matrix of the complex impedances, $\hat{U}$ is the vector of unknowns that merges each harmonic order, $\hat{F}$ is the expansion of the nonlinear force on each harmonic order, $\hat{F}_E$ is the expansion of the excitation force. The Harmonic Balance Method is easy to use when **F** is mathematically regular. The problem is nonlinear but algebraic and it can be solve by a Newton-Raphson algorithm. When it is singular, the Alternating Frequency-Time (AFT) method can allow to solve the problem. The main idea of AFT is to evaluate the nonlinear force in the time-domain and to make the harmonic balance in the frequency domain. This allows to solve problems with contact and friction for which the nonlinear force is non regular. Nevertheless, unfortunately, AFT becomes too expensive to solve large scale problems due to the computation of the jacobean matrix $\mathbf{J}^{(k+1)}$ and the computation of the non-linear force $\mathbf{F}^{(k+1)}$. To overcome, this problem, the domain can be decomposed in two subdomains Ω_S and Ω_J, see Figure 1. This leads to two systems of equations that share an interaction force **R** and the displacement filed $\mathbf{U_I}$ on Γ_I. **R** and $\mathbf{U^I}$ allow to couple both systems.

$$\begin{bmatrix} \mathbf{M}^S_{LL} & \mathbf{M}^S_{LI} \\ \mathbf{M}^S_{IL} & \mathbf{M}^S_{II} \end{bmatrix}\begin{bmatrix} \ddot{\mathbf{U}}^S_L \\ \ddot{\mathbf{U}}_I \end{bmatrix}+\begin{bmatrix} \mathbf{K}^S_{LL} & \mathbf{K}^S_{LI} \\ \mathbf{K}^S_{IL} & \mathbf{K}^S_{II} \end{bmatrix}\begin{bmatrix} \mathbf{U}^S_L \\ \mathbf{U}_I \end{bmatrix}=\begin{bmatrix} \mathbf{F}^S_E\cos(\Omega t) \\ \mathbf{R} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{M}^J_{JI} & \mathbf{M}^J_{JL} \\ \mathbf{M}^J_{LI} & \mathbf{M}^J_{LL} \end{bmatrix}\begin{bmatrix} \ddot{\mathbf{U}}_I \\ \ddot{\mathbf{U}}^J_L \end{bmatrix}+\begin{bmatrix} \mathbf{K}^J_{JI} & \mathbf{K}^J_{IL} \\ \mathbf{K}^J_{LI} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{U}_I \\ \mathbf{U}^J_L \end{bmatrix}+\begin{bmatrix} \mathbf{0} \\ \mathbf{F}^J(\mathbf{U}^J_L,\dot{\mathbf{U}}^J_L) \end{bmatrix}=\begin{bmatrix} -\mathbf{R} \\ \mathbf{0} \end{bmatrix} \tag{7}$$

To solve this problem, we use a fixed point algorithm, that alternatively solve a linear problem in the frequency domain on Ω_S in order to get the kinematic field $\mathbf{U^S}$,$\mathbf{U^I}$ when **R** and $\mathbf{F_E}$ are applied and a nonlinear quasi static problem in the time domain on Ω_J in order to get the kinematic field $\mathbf{U^J}$ and the reaction force **R** when $\mathbf{U^I}$ is applied. In order to make the problem easier to solve, one assume that the inertia terms are applied as a known field that comes from the previous increment, see [5] for more explanations.

3 MODEL REDUCTION

As seen before, the spatial decomposition of the problem associated with a fixed point algorithm and the Harmonic Bal- ance method allow to define a framework to compute the vibration levels of jointed structures under harmonic loadings. Nevertheless, on each subdomains, the problems involve a huge number of degrees of freedom. In order to compute the solution efficiently, one must use model order reduction techniques. Two reduction techniques are proposed here:

- the first one in order to reduce the number of DOFs, as many previous works [3], use the Craig-Bampton method to build a superelement of the linear part of the model;
- the second one, is the Principal Joint Strain Basis based on Dissipated Energy.

In a previous work, Festjens et al. [4] proposed the Principal Joint Strain Basis (PJSB) as a Model Order Reduction method for the joint domain. The idea is to find a very limited number of loadings **R** or boundary conditions $\mathbf{U_I}$ that span most the loadings induced by the vibration modes of the whole structure. If the PJSB exists, it allows to build meta-models for the behaviour of the structure. Such meta-models are often represented as mechanical charts, see [1].

To build the metamodels, a linear analysis is firstly performed on the global structure (full stick - interface tied) to determine the response function without any couplings. The basis Φ associated to the response function is normalized to stiffness matrix to give each eigenvectors the same strain energy. In order to reduce the joint model, one restrict the study to the joint:

$$\mathbf{U_I} = \Phi_\mathbf{I}\mathbf{Q} \tag{8}$$

with $\mathbf{U_I}$, the displacement field on the interface, and $\Phi_\mathbf{I}$, the truncated modal basis restricted to the interface area. In order to build detailed model of the joint, we introduce the Principal Joint Strain Basis Dissipated Energy (PJSBDE) as the optimal ritz basis to span the displacement field in the joint interface. To compute this basis, we build an energy indicator, The difference is obtained by observing two configurations of the joint - one which will totally sticking in the contact area, the other rather slippery in the interface (Eq. 9). It is therefore assumed that the strain energy difference is representative of the energy dissipation.

$$E_d = \Phi_\mathbf{I}^\mathbf{T}\left(\mathbf{K^J} - \tilde{\mathbf{K}}^\mathbf{J}\right)\Phi_\mathbf{I} \tag{9}$$

where E_d is the strain energy coupling matrix , $\Phi_\mathbf{I}$ the eigenmodes matrix, $\mathbf{K^J}$ is the stiffness matrix of the Ω_{NL} domain when the interface is tied, $\tilde{K}^J$ is the stiffness matrix of the ΩNL domain when the normal load is very low (bolt loosened). To compute the PJSBE, the idea is to simplify the basis $\Phi_\mathbf{I}$ since there must be redundancy between the eigenmodes as they have been considered locally. To achieve this goal, we extract the eigenvalues of the E_d . They are sorted and the eigenvector associated with the greatest one is kept as the boundary conditions $U_\mathbf{I}$. The meta model is built using these boundary conditions and making the associated generalized varying over the range of loading we want to study. The result is used to define the loadings we apply to a very detailed Non linear Finite Element model of the joint. The reaction forces, *R*, on the boundaries are extracted and used to solve the dynamic equation on Ω_L domain using Harmonic Balance Method

4 CONCLUSIONS

The theoretical frame proposed in this paper allows to simulate at a lower cost the dynamic behavior of the assembled structures. It is based on the use of methods of Ritz and meta - model. The latter are really interesting if several modes load the connection in the same way or if several connections are loaded in the same way.

REFERENCES

[1] A Caignot, P Ladeveze, D Néron, and J-F Durand. Virtual testing for the prediction of damping in joints. Engineering Computations, 27(5):621–644, 2010.

[2] TM Cameron and JH Griffin. An alternating frequency/time domain method for calculating the steady-state response of nonlinear dynamic systems. *Journal of applied mechanics*, 1989.

[3] M Claeys, JJ Sinou, JP Lambelin, and R Todeschini. Modal interactions due to friction in the nonlinear vibration response of the a harmony test structure: Experiments and simulations. Journal of Sound and Vibration, 376:131–148, 2016.

[4] H Festjens, G Chevallier, and JL Dion. Nonlinear model order reduction of jointed structures for dynamic analysis. Journal of Sound and Vibration, 333(7):2100–2113, 2014.

[5] Hugo Festjens, Gaël Chevallier, and Jean-luc Dion. A numerical tool for the design of assembled structures under dynamic loads. International Journal of Mechanical Sciences, 2013.

[6] Pierre Ladevèze, J-C Passieux, and David Néron. The latin multiscale computational method and the proper generalized decomposition. Computer Methods in Applied Mechanics and Engineering, 199(21):1287–1296, 2010.