GEOMETRIC NONLINEARITIES EFFECT ON CABLE LINEAR VIBRATIONS

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ABSTRACT

The present work investigates the free undamped vibrations of arbitrarily sagged cables according to the catenary theory. Defining the dynamic equilibrium configuration around the catenary static profile, an exact solution of the free linear transverse vibrations is developed analytically. The effectiveness of the established model is shown by means of comparisons between with results determined by classic formulations in case of horizontal and inclined shallow/non-shallow cables.

1 INTRODUCTION

The dynamic motion of cables is mainly studied with respect to both parabolic and the catenary static profiles. Based on the first works of Irvine [1] dedicated to the free linear oscillations of suspended cables, several research were developed according to the parabolic approach. Nevertheless, the necessity to account the catenary effect has been demonstrated for different types of cables with important sag as those used in suspended bridges or as transmission lines. Accordingly, the catenary model has been adopted in some recent papers: while an analytical solution specific to the transversal motion was proposed by Lacarbonara et al. [2] by considering the exact nonlinear static profile of horizontal non-shallow cables, Zhou et al. [3] has solved the in-plane dynamic problem specific to taut inclined cables by introducing the cubic approximation of the catenary geometry. In the light of previous models, the present work provides accurate analytical solution based on the elastic catenary theory and related to the free linear vibrations of both horizontal and inclined cables.

2 ANALYTICAL SOLUTION TO THE CABLE DYNAMIC PROBLEM

A suspended cable between two fixed supports A and B displayed in Figure 1 is characterized by a specific weight γ_c, a non deformable cross-section denoted by A_c and a linearly elastic material defined by a Young elastic modulus E_c. A local Cartesian coordinate system (x, y, z) is attached to the cable's chord having an angle α with respect to l defining the horizontal projection of the chord cable length L.

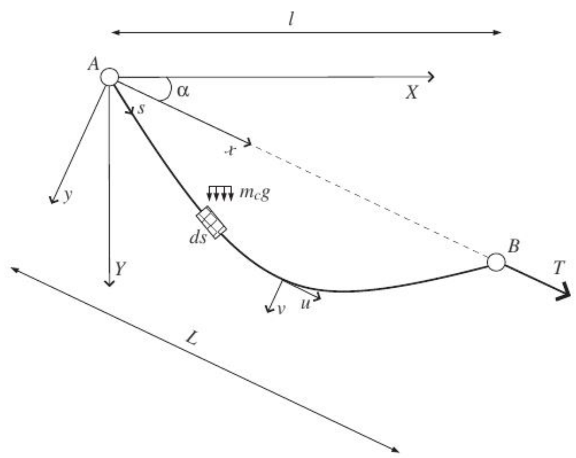

Figure 1. Static equilibrium configuration of a suspended cable

Under the action of its total weight, the strained static profile is determined expressed as follows:

$$\begin{cases} \dfrac{y(x,\tau)}{L} = \dfrac{\cosh(C_1(\tau)) - \cosh\left(C_1(\tau) - \tau\frac{x}{L}\right)}{\tau} \quad ; \quad \tau = \dfrac{m_c g L}{T} \\ C(\tau) = \dfrac{\tau e^{\tau}}{e^{\tau}-1}\tan\alpha \quad ; \quad C_1(\tau) = \ln\left(C + \sqrt{C^2 + e^{\tau}}\right) \end{cases} \tag{1}$$

Considering the static equilibrium configuration defined by the catenary profile given previously and taking into account the assumptions related to the linear vibration theory, the cable dynamic motion reduces to the transversal component characterized by dimensionless frequencies Ω_k

obtained as roots of the following transcendental equation:

$$\tan\Omega_k\left[\frac{\left(\Omega_k^2+\tau^2\right)^2}{\lambda_c^2}-\frac{I}{\tau L}\left(\Omega_k^2+\tau^2\right)-\rho_4\tau\right]+2\rho_3\tau\sin\Omega_k-\rho_1\Omega_k+\frac{\rho_2}{\cos\Omega_k}\Omega_k=0 \quad (2)$$

with:

$$\begin{cases} I=\dfrac{L}{2}+\dfrac{L}{4\tau} \quad ; \quad \rho_1=\cosh^2 C_1+\cosh^2\left(C_1-\tau\right) \quad ; \quad \rho_2=\cosh C_1\cosh\left(C_1-\tau\right) \\ \rho_3=\cosh C_1\sinh\left(C_1-\tau\right) \quad ; \quad \rho_4=\dfrac{\sinh\left(2\left(C_1-\tau\right)\right)+\sinh\left(2C_1\right)}{2} \quad ; \quad \lambda^2=\dfrac{\tau^2}{\eta\chi} \end{cases} \quad (3)$$

where λ is the Irvine parameter depending on the dimensionless thrust $\eta=T/E_cA_c$ and the cable's curvature χ tending to unity in case of horizontal cables according to Irvine formulation [1] based on parabolic approach. However, accurate expression of the curvature term is obtained according to the actual formulation for both horizontal and inclined arbitrarily sagged cables:

$$\chi=\frac{\sinh\left(\frac{3\tau}{2}\right)\cosh\left(3\left(C_1-\frac{\tau}{2}\right)\right)+9\sinh\left(\frac{\tau}{2}\right)\cosh\left(C_1-\frac{\tau}{2}\right)}{6\tau} \quad (4)$$

It must be noted that equation (4) reduces to the formula proposed by Lacarbonara et al. [2] for horizontal non-shallow cables given by:

$$\chi=\frac{9\sinh\left(\frac{\tau}{2}\right)+\sinh\left(\frac{3\tau}{2}\right)}{6\tau} \quad (5)$$

3 MODEL VALIDATION

	τ	Irvine's	Enhanced Irvine's	Present	Exact
	1.5	8.95	8.79	8.48	8.44
Error(%)		6.04	4.15	0.47	
	2.5	8.95	8.66	7.85	7.63
Error(%)		17.30	13.50	2.88	

Table 1: Lowest symmetric frequencies of horizontal cables with $\lambda=10\pi$ obtained with Irvine theory, enhanced Irvine theory, and present model, and relative errors (%) with respect to the exact (non-condensed) model.

In order to show the accuracy of the proposed solution, an investigation is performed regarding the evaluation of dimensionless frequencies obtained according to the actual formulation from one side and using models found in th literatue. The results related to the comparison held on non-shallow horizontal profiles and taut inclined cables are respectively reported in Table 1 and illustrated by Figure 2. As it may be remarked from Table 1, the error made by the general catenary-based model is by far smaller than those inherent to proposed solutions by Irvine [1] and Lacaronara et.[2]. In fact, the difference between present results and exact ones ranges from 0.47% to 2.88%, with an expected increase for the looser cable, where the effect of the longitudinal dynamics (herein neglected) becomes more important; however, it increases more for both the enhanced (from 4.15% to 13.5%) and the original Irvine theory (6.04% to 17.30%). On the other side, the validity of the present model is demonstrated by the the Figure 2 specific to the case of taut inclined cables. As a matter of fact, the absolute relative error with respect to the frequencies found by the Galerkin (resp. Zhou) varies from 0.03% (resp. 0.07%) to 1.202% (resp. 1.47%) for an inclination $\alpha = 10^{\circ}$ and ranges from 0.05% (resp. 0.048%) to 0.79% (resp. 0.95%) when $\alpha = 60^{\circ}$. It must be noted that the results obtained analytically remain acceptable since the absolute relative error with respect to Zhou's results - which are nearly concident with the "exact" ones - does not exceed about 1.5%: such small error is likely due to the factor of the weight component parallel to the cable chord accounted in Zhou's model and neglected in the present formulation.

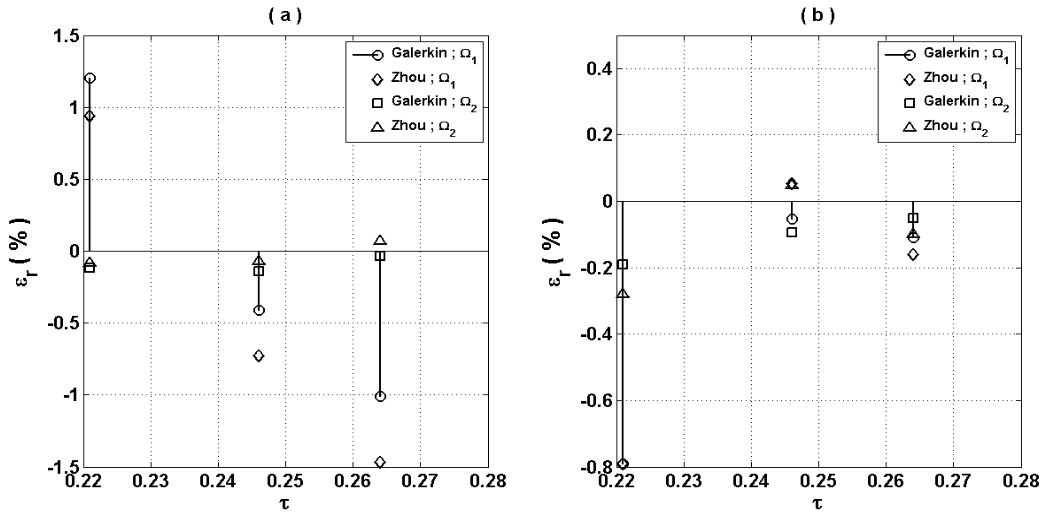

Figure 2: Relative errors $\epsilon_r(\%)$ related to the 1^{st} and 2^{nd} dimensionless frequencies $\Omega_{1,2}$ with respect to results obtained by both Galerkin and Zhou's methods and presented in [3] for inclined cables with: (a) $\alpha = 10^{\circ}$; (b) $\alpha = 60^{\circ}$

4 CONCLUSION

A general catenary-based model is developed analytically for the transverse linear free undamped vibrations of shallow/non shallow arbitrarily inclined cables. The exactitude of the proposed solution is exhibited by a maximum absolute relative errors $|\epsilon_r| = 2.88\%$ and $|\epsilon_r| \simeq 1.5\%$ calculated with respect to the exact results respectively for non-shallow horizontal cables ($\tau = 2.5$) and taut inclined cables.

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